ROTATIONAL MOTION

* TRANSLATIONAL MOTION:

→ It is the motion attend by a body, if the velocity of all the particle of the body is the same at any instant of motion.

→ Example-

(i) The motion of a car along straight line.

(ii) Motion of train on it track.

(iii) A person walking on a road.

(iv) A flying bird in a sky.

→ There are three type of translational motion.

① LINEAR OR ONE-DIMENSIONAL MOTION

→ The motion in which the body moves in a straight line is called linear or one dimensional motion.

→ Example - (i) Freely falling body.

(ii) Running bus on a straight road.

② TWO-DIMENSIONAL MOTION

→ when a body is moving in a plane then two coordinate changes either (x,y) or (y,z) or (z,x)

→ Example:-

(i) Motion of a car on zig zag road

(ii) Projectile motion.

③ THREE-DIMENSIONAL MOTION

→ when a body is moving in the sky then all the three coordinate changes and this type of motion is called three dimensional motion.

→ Example:
(i) Bird flying in the sky.
(ii) An aeroplane in the sky.

* ROTATIONAL MOTION

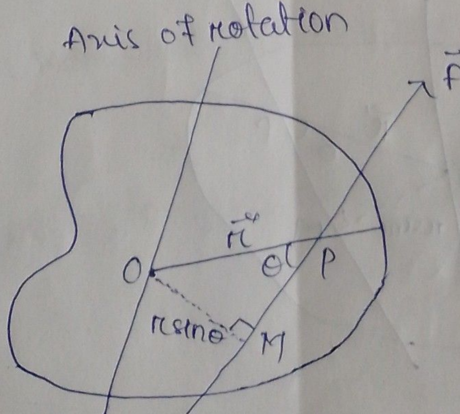
→ When a rigid body ~~rotat~~ rotates around a fixed axis, then each point of the body move in a circle whose centre lies on the axis of rotation & each point traces the same angle in a particular time interval. Such motion is called rotational motion.

→ The motion of any object in which every part of the object rotates about a common axis in a circular path is known as rotational motion.

→ Example-
(i) The motion of the wheels in the vehicle.
(ii) Rotation of earth about it axis.
(iii) A working ceiling fan.
(iv) The motion of minute & hour hand of a watch.

* TORQUE (τ)

→ The torque produce by a force about an axis is equal to the product of magnitude of force and its perpendicular distance from the axis of rotation to the line of action of force.



→ let a force (F) acting on a point p of a rigid body & the axis rotation possess through the origin (O). $\vec{r}$ is the position vector of point 'p'. then the rotation of the body due to the torque exerted by a force (F) is given by

→ Torque = force × Perpendicular distance of the line of action of a force from the axis of rotation.

→ Here OM is the perpendicular drawn on the line of action of force from the origin.

- <u>In triangle OMP.</u> $\sin\theta = \frac{OM}{OP}$

$\Rightarrow OM = OP \sin\theta$

$\Rightarrow OM = r\sin\theta$

Hence $\tau = F \times OM$

$\Rightarrow \tau = F r \sin\theta$

$\Rightarrow \tau = r F \sin\theta$

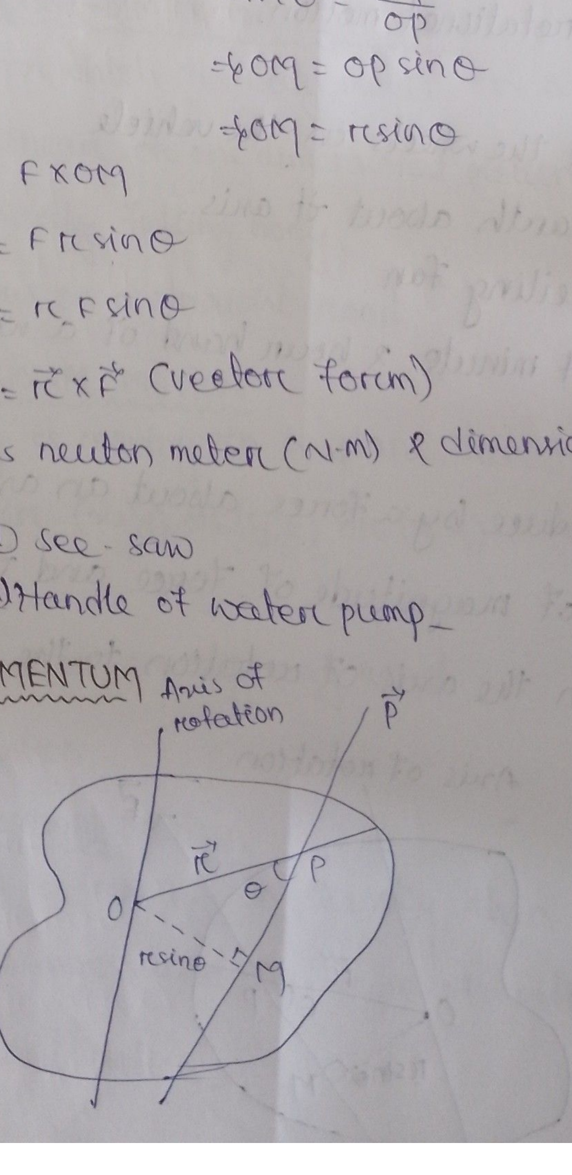
$\Rightarrow \tau = \vec{r} \times \vec{F}$ (vector form)

→ It si unit is neuton meter (N-m) & dimensional : $[M^1 L^2 T^{-2}]$

<u>Example :-</u> (i) see-saw

(ii) Handle of water pump

<u>ANGULAR MOMENTUM</u>



→ it is define as the product of linear momentum and the perpendicular distance of linear momentum from the axis of rotation.

→ A linear momentum $\vec{p}$ is applied at point P. The position vector of point 'p' is $\vec{r}$ from the origin, so angular momentum is given by.

$$\Rightarrow L = P \times OM.$$

<u>In triangle OPM</u> $\Rightarrow \sin\theta = \frac{OM}{OP}$

$$\Rightarrow OM = OP\sin\theta$$

$$\Rightarrow OM = r\sin\theta$$

So, eq. ① become $\Rightarrow L = P \times r\sin\theta$

$$\Rightarrow L = Pr\sin\theta$$

$$\Rightarrow L = \vec{r} \times \vec{p} \text{ (vector form)}$$

→ it si unit is $kg\,m^2/sec$.

→ <u>Examples</u>:-

(i) Orbital angular momentum of earth due to it revolution about the sun.

(ii) Spin angular momentum of the earth due to its rotation about it axis.

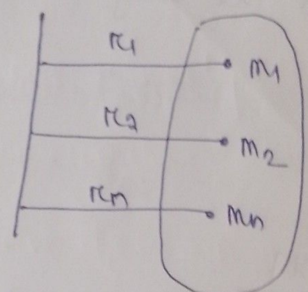
(iii) Spin angular momentum of rotation of chair about it axis.

✱ Relation between torque and angular momentum

$$\Rightarrow \tau = \frac{dL}{dt}.$$

MOMENT OF INERTIA

→ Moment of inertia of a particle is define as the product of mass of the particle & the square of the distance of the particle from the axis of rotation.



→ if a body contain 'n' number of particle, then

$$\Rightarrow I = M_1 r_1^2 + M_2 r_2^2 + M_3 r_3^2 + \cdots + m_n r_n^2$$

$$\Rightarrow I = \sum_{i=0}^{n} M_i r_i^2$$

→ It si unit is $kg\,m^2$

NOTE :- Moment of inertia is a property of a body due to which it opposes any change in its state of rest or uniform rotation.

RELATION BETWEEN TORQUE AND MOMENT OF INERTIA

we know torque, $\tau = \vec{r} \times \vec{F}$

$$\Rightarrow \tau = rF\sin\theta$$

if $\theta = 90°$, that is position vector ($\vec{r}$) is perpendicular to the force (F).

then, $\Rightarrow \tau = rF$

$$\Rightarrow \tau = rma$$

$$\Rightarrow \tau = rm(r\alpha)$$

$$\Rightarrow \tau = mr^2\alpha$$

→ If the body having n number of particle then the net torque is given by.

$$\Rightarrow \tau_{net} = \sum_{i=1}^{n} m_i r_i^2 \alpha$$

$$\Rightarrow \tau_{net} = I\alpha$$

or $\vec{\tau}_{net} = \sum \vec{r}_i \times \vec{F}_i$

RELATION OF ANGULAR MOMENTUM AND MOMENT OF INERTIA

$$\Rightarrow L = \vec{r} \times \vec{p}$$

If $\theta = 90°$, then $L = rp \sin\theta$

$$\Rightarrow L = rp$$

$$\Rightarrow L = rmv \quad \because p = mv$$

$$\Rightarrow L = rmr\omega \quad \because r\omega = v$$

$$\Rightarrow L = mr^2\omega$$

$$\Rightarrow L = I\omega$$

or $\Rightarrow \vec{L}_{net} = \sum_{i=1}^{n} \vec{r}_i \times \vec{P}_i$

CONSERVATION OF ANGULAR MOMENTUM

statement - If no external torque is applied on a body rotating about an axis, then the total angular momentum of the body remain constant.

We know, $\tau = \frac{dL}{dt}$

If $\tau = 0$, $\frac{dL}{dt} = 0$, then L is constant

Further $L = I\omega$

then $I\omega$ is also constant. (for $\tau = 0$)

Example:-

(i) when a person stands on a ~~to~~ turn table with outstretched arm and holds weight in each hand, The turntable revolves with certain angular momentum. Now if we pulls the weight towards his body, there is a sudden increase in angular velocity. Here no external torque exerted, so the conservation of angular momentum and the moment of inertia decrease the angular velocity must be increased.

(ii) Ballet dancer, divers are using the principle of conservation of angular momentum.

RADIUS OF GYRATION

→ The radius of gyration of rotating body is equal to the radial distance from the axis of rotation, the square of which multiplied by the total mass of the body gives the moment of inertia of that body.

Mathematically: $I = MK^2$

where, M = mass of the body

K = radius of gyration.

→ consider a body of mass (M), consisting of particles each of mass (m).

→ let ~~FE~~ $r_1, r_2, r_3 \text{------} r_n$ be the perpendicular distance of the particles from the axis of rotation.

So, $\Rightarrow I = mr_1^2 + mr_2^2 + \cdots + mr_n^2$

$\Rightarrow I = m(r_1^2 + r_2^2 + \cdots + r_n^2)$

$\Rightarrow I = MK^2$

Further $\Rightarrow MK^2 = m(r_1^2 + r_2^2 + \cdots + r_n^2)$

$$\Rightarrow MK^2 = m \times n \left(\frac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}\right)$$

$$\Rightarrow MK^2 = M\left(\frac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}\right)$$

$$\Rightarrow K^2 = \left(\frac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}\right)$$

$$\Rightarrow K = \sqrt{\left(\frac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}\right)}$$

$\Rightarrow K =$ root mean square distance.

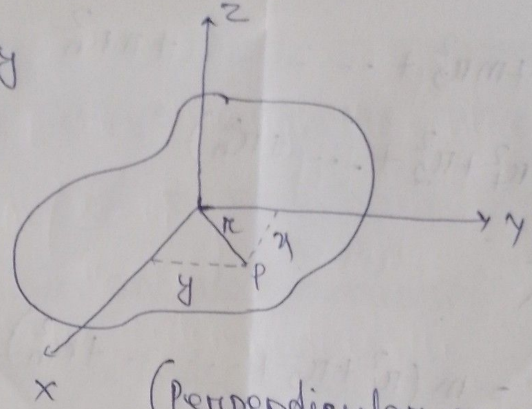
∴ The radius of gyration of a body about the given axis of rotation is the root mean square of the perpendicular distance of the particles from the axis of rotation.

THEOREM OF PARALLEL AND PERPENDICULAR AXES

A. PERPENDICULAR AXES THEOREM

$\Rightarrow$ This theorem state that the moment of inertia of lamina about an axis perpendicular to its plain (I_z) is equal to the sum of moment of inertia of the lamina about two mutually perpendicular axis (I_x and I_y) lie its plain and intersecting at a point where the perpendicular axis passes.

thus, $I_z = I_x + I_y$

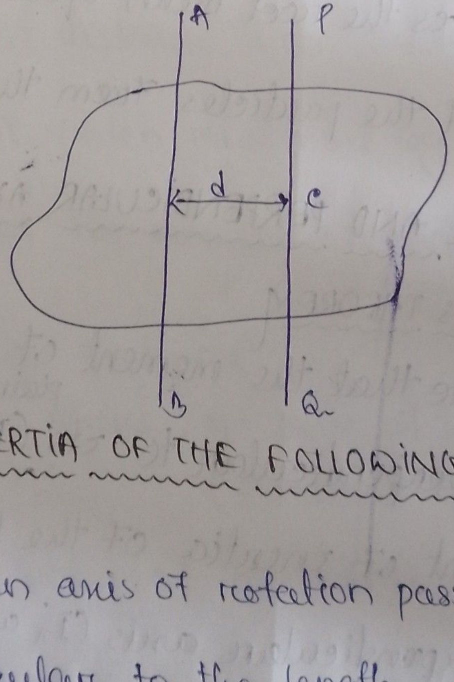


(Perpendicular axes theorem)

b. PARALLEL AXES THEOREM

This theorem states that the moment of inertia (I_{AB}) of a body about any given axis (AB) is equal to the sum of moment of inertia about a parallel axes passes through the centre of mass (c) of the body and product of mass (M) of the body and the square of distance (d) between the two parallel axis.

thus, $I_{AB} = I_c + Md^2$



MOMENT OF INERTIA OF THE FOLLOWING BODIES

(A) ROD :-

(i) M.I. about an axis of rotation passes through the centre and perpendicular to the length.

$$I = M\left(\frac{L^2}{12} + \frac{r^2}{4}\right)$$, where, M = mass
L = length
r = radius.

(ii) M.i. about an axis of rotation passes through one end and perpendicular to the length of rod, $I' = \frac{mr^2}{3}$

(iii) M.i. about an axis of the rod, $I = \frac{1}{2} mr^2$

(B) DISC :-

(i) M.i. about axis passes through centre and perpendicular to plain $= \frac{1}{2} mr^2$

(ii) M.i. about axis passes through centre parallel to plain or lies in plane or diameter $= \frac{1}{4} mr^2$

(iii) M.i. about axis is tangent and perpendicular to plain $= \frac{1}{2} mr^2 + mr^2 = \frac{3}{2} mr^2$

(iv) M.i. about the tangent, lies in plain $= \frac{1}{4} mr^2 + mr^2 = \frac{5}{4} mr^2$

(C) RING :-

(i) M.i. about axis passes through centre and perpendicular to plain $= Mr^2$

(ii) M.i. about axis passes through centre, lies in the plain or diameter $= \frac{1}{2} mr^2$

(iii) M.i. about an axis as tangent lies in plain $= \frac{1}{2} mr^2 + mr^2$
$= \frac{3}{2} mr^2$

(iv) M.i. about tangent and perpendicular to plain $= Mr^2 + Mr^2 = 2Mr^2$

(D) SPHERE :-

(i) M.i. about axis passes through the centre or about diameter $= \frac{2}{5} mr^2$

(ii) M.i. about tangent $= \frac{2}{5} mr^2 + mr^2 = \frac{7}{5} mr^2$

(E) HOLLOW SPHERE

(i) M.i. about axis passes through the centre or about diameter $= \frac{2}{5} M \left(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3} \right)$

where, M = mass, r_2 = outer radius, r_1

(F) SPHERICAL SHELL :

(i) M.i. about axis passes through centre or diameter

$= \frac{2}{3} Mr^2$