Work, Power & Energy

* **Work** :- work is said to be done if force is applied on a body, displaces the body the force has an component along the direction of displacement.

OR

→ work is also defined as the dot product of force and ~~dep~~ displacement.

Mathematically :- $\vec{W} = \vec{F} \cdot \vec{S}$

$\Rightarrow \vec{W} = FS\cos\theta$

where, $\vec{W}$ = work done.

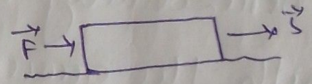
F = Force

s = displacement

θ = angle between force & displacement.

case-1 If $\theta = 0°$

then, $\vec{W} = FS\cos(0°)$



$\Rightarrow \vec{W} = FS(1)$

$\Rightarrow \vec{W} = FS$

→ Here force & displacement are in same direction & the work done up on the body. This means the work done is positive.

Example :- An object falling freely due to gravity.

case-2 If $\theta = 90°$

then, $\vec{W} = FS(\cos 90°)$

$\Rightarrow \vec{W} = FS(0)$

$\Rightarrow \vec{W} = 0$

→ Here the force & displacement are perpendicular to each other & work done is zero.

Example :- A person carrying a box over his head ~~wo~~walking in horizontal direction.

Case-3 $\theta = 180°$

then, $\vec{W} = FS(\cos 180°)$

$\Rightarrow \vec{W} = FS(-1)$

$\Rightarrow \vec{W} = -FS$

→ Here force & displacement are opposite direction & the work done is negative. This means the work done is negative.

Example:- work done by the force of friction.

→ SI unit of work is joules & cgs unit is ergs.

→ 1 joule is equal to 10^7 erg.

→ Dimensional formula of work = $[M^1 L^2 T^{-2}]$

Q// calculate the work done in dragging a block of 50m horizontally with a rope making angle of 30° with the ground with a force of 60N.

→ Given

Force = 60 N

displacement = 50 m

$\theta = 30°$.

so, $\vec{W} = FS\cos\theta$

$\Rightarrow \vec{W} = 60 \times 50 (\cos 30°)$

$= 3000 \times \frac{\sqrt{3}}{2}$

$= 1500 \times \sqrt{3}$ $\{\sqrt{3} = 1.732\}$

$= 2598$ J

Q// if a person uses 45N of forces to lift a shoot case & walk while doing work of 1950 J, then calculate the distance covered by the person with a shootcase

→ Given. Force = 45N
Work = 1250J
θ = angle will be 0°.

So, $\vec{W} = Fs\cos\theta$

$\Rightarrow 1250 = 45 \times s \times (\cos 0°)$

$\Rightarrow \frac{1250}{45} = s \times 1$

$\Rightarrow s = 27.78$ m.

* FRICTION :- The force which opposes or tends to oppose the relative motion between the two surface in contact is called force of friction.

→ There are three type of friction.

(i) static friction
(ii) Limiting friction
(iii) Kinetic (dynamic) friction.

* STATIC FRICTION :- This is the opposing force which exist between the surface & the body at rest.

Example :- A book on a table.

* LIMITING FRICTION :- The maximum value of static friction is called limiting friction.

→

* KINETIC FRICTION :- This is the opposing force which exist when two surface move over each other.

→ Eg :- walking on the road, pushing a chair.

* LAW OF LIMITING FRICTION :-

→ The direction of force of friction is always opposite to the direction of the motion of the object.

→ The force of limiting friction depend up on the nature & state of polish of the surface in contact.

→ it acts tangentially to the interface between two surface.

→ The magnitude of limiting friction (F_L) is directly proportional to the magnitude of normal reaction (R) between the two surface in contact. i.e. $F_L \propto R$.

→ The magnitude of limiting friction between two surface is independent of the area & shape of the surface in contact as long as the normal reaction remain the same.

* **COEFFICIENT OF FRICTION** :- it is defined as the ratio of frictional force to the normal reaction.

→ Mathematically, $\mu = F/R$

where, μ = coefficient of friction.
F = force of friction
R = Normal reaction.

Q// A box of masses 30 kg. is pulled on a horizontal surface by applying a horizontal force. If the coefficient of friction between box & surface is 0.25, find the force of friction exerted by the horizontal surface on the box.

⇒ Given, weight = 30 kg

we know that weight = Normal reaction = mg

$\Rightarrow R = 30\,kg \times 0.98 = 294.0$

coefficient of friction = 0.25

so, force of friction = $\mu \times R$

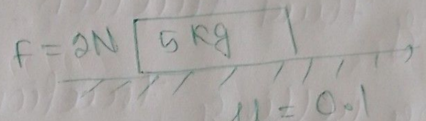
$= 0.25 \times 294.0$

$= 73.230$ N.

Q/ Find the force of friction, take $g = 10 m/s^2$

(I) limiting friction

(II) static friction



⇒ Given force = 2N

Mass = 5 kg

so, normal reaction = mg = 5 × 10 = 50

μ (coefficient of friction) = 0.1

then, force of friction, = $\mu \times R$

$= 0.1 \times 50$

$= 5$ N

Hence force of friction = 5N

∴ (I) limiting friction is 5N

(II) static friction is 2N, as 2N of force is applied on the body (According to the third law of newton)

* **REDUCING FRICTION AND ITS ENGINEERING APPLICATION**

Methodes of reducing the friction includes

(I) Making smoother surface by different methodes like grinding, robbing, polishing, chemical etching etc.

(ii) using semi solid paste like lubricants, in the metal parts used in heavy machineries.

(iii) Making streamlined or aerodynamic body of bullet trains, cars etc.

(iv) Reduction in pressure or weight of the object. This reduce the wear & tear of the vehicle tyres.

(v) Reducing the contact between surface of two object.

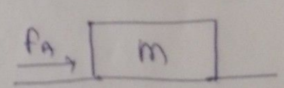
(vi) using fluid friction.

(vii) using rolling friction instead of sliding friction, it can be done by using ball bearing

* **WORK DONE AGAINST FRICTION WITH RELATED FRICTION.**

FORCE OF FRICTION ON HORIZONTAL SURFACE

→ Assume a box of masses 'm' is lying on the horizontal surface of the table.



→ A force (F_A) is applied to the box which tries to move the box. But due to the force of static friction (F_s) the box does not move.

Hence, $F_s = \mu_s R$

$\Rightarrow F_s = \mu_s mg \quad \{\because R = mg\}$

→ suppose the external force (F_A) Applied on the box is large enough & displaces the box. The box starts moving with an acceleration (a) and experience the force of kinetic friction (F_k)

Hence the net force acting on the box is given by.

$$ma = F_A - F_K$$

$$\Rightarrow ma = F_A - \mu_K mg$$

$$\Rightarrow a = \frac{F_A - \mu_K mg}{m}$$

$$\Rightarrow a = \frac{F_A}{m} - \mu_K g$$

And the work done of the friction is given by,

$$\vec{W} = f_K \cdot r\,(\cos\theta) \qquad \left\{ \begin{array}{l} r = \text{displacement} \\ \theta = 0^\circ \end{array} \right\}$$

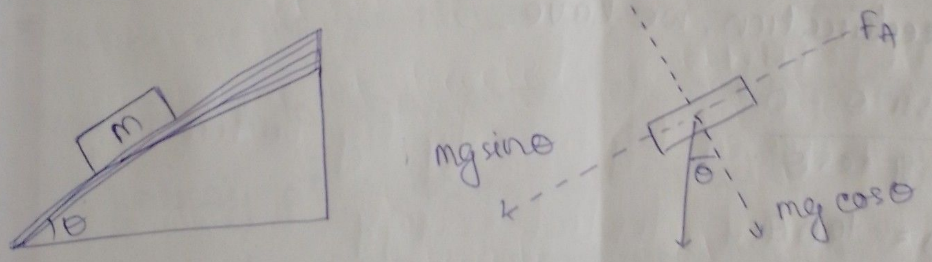
$$\Rightarrow \vec{W} = f_K r\,(\cos 0^\circ)$$

$$\Rightarrow \vec{W} = f_K r\,(1)$$

$$\Rightarrow \vec{W} = f_K r.$$

FORCE OF FRICTION ON INCLINED PLANE (SURFACE)

- Consider a block is kept motionless on an inclined plane which make an angle θ with the horizontal.



- Here the weight of the block has two components, one parallel to the surface of inclined plane and another perpendicular to surface of inclined plane, so.

so, $W_\perp = mg\cos\theta = N$

And we know $\mu_s = \frac{f_s}{N}$

$$\Rightarrow f_s = \mu_s N$$

$$\Rightarrow f_s = \mu_s mg\cos\theta$$

Now, $W_{\parallel} = mg\sin\theta$

suppose the block start moving downward with an acceleration "a", so the kinetic force of friction is given by $f_K = \mu_K N$

$$\Rightarrow f_K = \mu_K mg\cos\theta$$

From newton's second law of motion, we have -

$$ma = mg\sin\theta - f_K \qquad \{ma = f_A - f_K\}$$

$$\Rightarrow ma = mg\sin\theta - mg\cos\theta\,\mu_K$$

$$\Rightarrow \frac{-ma + mg\sin\theta}{mg\cos\theta} = \mu_K$$

$$\Rightarrow \mu_K = \frac{mg\sin\theta - ma}{mg\cos\theta}$$

$$\Rightarrow \mu_K = \frac{\cancel{m}(g\sin\theta - a)}{\cancel{m}g\cos\theta}$$

$$\Rightarrow \mu_K = \frac{g\sin\theta - a}{g\cos\theta}$$

For zero acceleration, we have -

$$\Rightarrow \mu_K = \frac{g\sin\theta - 0}{g\cos\theta}$$

$$\Rightarrow \mu_K = \mu_s = \tan\theta$$

Q) A skier of 60 kg rest on an inclined surface of the mountain. The inclination of angle θ is 30° with the horizontal & the skier just begins to slide, what is the coefficient of static friction between the skier & the surface of mountain? If the friction is 50N. find the coefficient of kinetic friction

→ Given :

weight of skier = 60 kg.

angle of inclination = 30°

force of friction decreasing slide is 50 N

so, ~~static friction (~~

coefficient of static friction (μ_s) = $\tan\theta$

$= \tan 30° = \frac{1}{\sqrt{3}} = \frac{1}{1.732} = 0.577$ N

& coefficient of kinetic friction (μ_k) = $\frac{f_k}{N}$

$= \frac{f_k}{mg\cos\theta}$

$= \frac{50\text{ N}}{60 \times 9.8 \times \cos 30°}$

$= 0.098$ ~~N~~ Ans

ENERGY :

- Energy is the quantitative physical ~~qua~~ property that is transferred to some object for carrying out work.
- The energy of a body is defined as the capacity of doing work and is measured by the amount of work a body can do.

→ SI unit of energy is Joules (J)

→ It is a scalar quantity.

→ It dimensional formula is $[M^1 L^2 T^{-2}]$

Some practical units of energy & their equivalence to joule is given by.

Sl. No.	UNIT	SYMBOL	EQUIVALENCE
1	Kilowatthour	kWh	3.6×10^6 J
2	calorie	cal	4.2 J
3	Erg	erg	10^{-7} J
4	Electron volt	EV	1.6×10^{-19} J

KINETIC ENERGY :-

→ The energy possessed by a body by virtue of its motion is called kinetic energy.

→ When an object of mass "m" is in motion with some velocity 'v'. it is said to be possess kinetic energy.

→ Hence :- K.E. = ½ mv^2

Example :- (i) The kinetic energy of running water is used to run the water mills.

(ii) Kinetic energy of hammer is used to drive a nail in wood.

(iii) A bullet fired from a pistol can penetrate in to its target because of its kinetic energy –

(iv) fast moving ball thrown by a pace baller –

~~Pote~~ POTENTIAL ENERGY :-

→ The energy possessed by a body by virtue of its position is called potential energy.

→ it is a stored energy in an object

→ Hence :- P.E = mgh –

Examples :-

(i) ~~Enne~~ Energy possessed by a compressed spring.

(ii) Potential energy of streched bow is used to give high velocity to arrow.

(iii) Potential energy of water is used to get electrical energy at dam.

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→ Potential energy of three types.

(i) Gravitational potential energy.

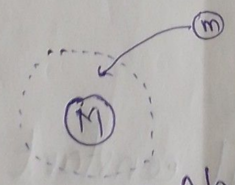
(ii) Elastic potential energy.

(iii) Electrostatic potential energy.

GRAVITATIONAL POTENTIAL ENERGY :-

→ Meaning :- The potential energy possessed by a body by virtue of its position with respect to earth.

→ Def :- when an object of mass 'm' is brought from infinity to a point inside the gravitational field of some source mass (M) with constant speed, the amount of work done in displacing the object in to the gravitational field of source mass is stored in the form of potential energy and is known as gravitational potential energy.



→ it is given by $U = mg\Delta h$

where, m = mass of the object

g = accelaration due to gravity.

Δh = height of object from ground.

Example :-

(I) water stored in dam at height 'h'.

(ii) swimmer on diving board before the jumps in to the swimming pool.

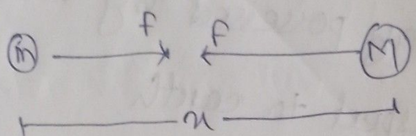
(iii) Any fruit on the tree - before it falls.

(iv) Thick layer of ice at the peak of the himalayas.

(v) A cleaning worker on the tall building.

NOTE

G → universal gravitational constant.



$F \propto Mm$

$F \propto \frac{1}{x^2}$

then, $F \propto \frac{Mm}{x^2}$

$\Rightarrow F = G\frac{Mm}{x^2}$

$\Rightarrow G = \frac{Fx^2}{Mm}$

If $M = 1$ unit

$m = 1$ unit

$x = 1$ unit

then, $G = F$

∴ universal gravitational constant is a force.

DERIVATION OF GRAVITATIONAL POTENTIAL ENERGY :-

Let the source mass be (M). initially the test mass (m) is placed at infinity (∞). Under the influence of constant gravitational field or force (F) of source mass (M), test mass (m) is displace by a very small amount (dx) & a small amount of work (dw) is done on the test mass (m). This is given by

$$dw = F \cdot dx$$

$$\Rightarrow dw = -G \frac{Mm}{x^2} dx$$

(The gravitational force (F) is attractive & the displacement of the test mass is toward the source mass, that is in opposite direction)

Integrating the equation on both side

$$\Rightarrow \int_{\infty}^{r} dw = \int_{\infty}^{r} -G \frac{Mm}{x^2} dx$$

$$\therefore \left\{ \int x^n dx = \frac{x^{n+1}}{n+1} \right\}$$

$$\Rightarrow [w]_{\infty}^{r} = \left[G \frac{Mm}{x} \right]_{\infty}^{r}$$

$$\Rightarrow [w]_{\infty}^{r} = \frac{GMm}{r} - \frac{GMm}{\infty} \qquad \therefore \left\{ \text{as } \frac{1}{\infty} = \text{not defined} \right\}$$

$$\Rightarrow w = \frac{GMm}{r}$$

Since the work done ~~on~~ on the test mass is stored as its potential energy (U). so we can write.

$$U = -\frac{GMm}{r}$$

This is gravitational potential energy of a test mass stored at a point with distance (r) from the source mass (M).

Now, if the test mass moves inside the gravitational field of source mass (M) from one point (A) to an other point (B).

- Let r_i be the position of test mass for point (A) from the source mass (M) & r_f be the position of the test mass from the source mass (M)

- Then the change in gravitational potential energy is given by.

$$\Delta U = -\left[\frac{GMm}{r}\right]_{r_i}^{r_f}$$

$$\Rightarrow \Delta U = -\left[\frac{GMm}{r_f} - \frac{GMm}{r_i}\right]$$

$$\Rightarrow \Delta U = \frac{GMm}{r_i} - \frac{GMm}{r_f}$$

$$\Rightarrow \Delta U = GMm\left[\frac{1}{r_i} - \frac{1}{r_f}\right]$$

if $r_i = R$ & $r_f = R+h$

then, $\Delta U = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$

$$\Rightarrow \Delta U = GMm\left(\frac{h}{R(R+h)}\right)$$

Since, $h <<<< R$

so, $(R+h) \sim R$

Hence, $\Delta U = GMm\left(\frac{h}{R(R)}\right)$

$$\Rightarrow \Delta U = GMm\left(\frac{h}{R^2}\right)$$

$$\Rightarrow \Delta U = mgh \quad \therefore \left(g = \frac{GM}{R^2}\right)$$

Note

$F = G\frac{Mm}{r^2}$

$\Rightarrow mg = G\frac{Mm}{r^2}$

$\Rightarrow g = \frac{GM}{r^2}$

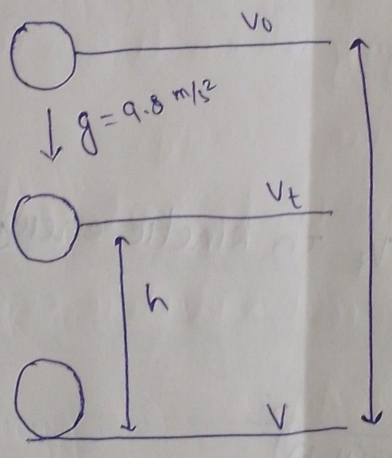
MECHANICAL ENERGY

→ Mechanical energy of an object is the sum of the kinetic energy & the potential energy.

→ i.e. Mechanical energy = kinetic energy + potential energy

→ Mechanical energy describe the motion or position or both of an object.

CONSERVATION OF MECHANICAL ENERGY FOR A FREELY FALLING BODY :-



→ The total mechanical energy of a system is conserved, i.e. the energy can niether be created nor destroyed, it can only be converted from one form to another form provided the forces acting to do some work are conservative force.

(i) At height (H)

→ Let us consider a ball of mass (m) is dropped from a height (H)

→ Here, total mechanical energy

$= \text{kinetic energy} + \text{potential energy}.$

$= 0 + mgH$

$= mgH$

$$\left\{ \begin{array}{l} \text{As velocity} = 0 \\ K.E = \frac{1}{2}mv^2 = \frac{1}{2}m0^2 = 0 \\ P.E = mgH \end{array} \right\}$$

(ii) At height (h)

Here the potential energy $= mg(H-h)$

kinetic energy $= \frac{1}{2}mv_t^2$

using the equation of motion, under gravity.

$\Rightarrow v_t^2 - u^2 = 2gs \qquad \{v^2 - u^2 = 2as\}$

$\Rightarrow v_t^2 - 0 = 2gs$

$\Rightarrow v_t = \sqrt{2g(H-h)}$

Putting the value of v_t in kinetic energy

we get, $K.E = \frac{1}{2}mv_t^2$

$= \frac{1}{2}m\left(\sqrt{2g(H-h)}\right)^2$

$= \frac{1}{2}m\,2g(H-h)$

$= mg(H-h) = mgH - mgh$

Now total mechanical energy

$= \text{Kinetic energy} + \text{Potential energy}$

$= mgH - mgh + mgh$

$= MgH$

(iii) At height zero :-

Here height is zero, so potential energy = $mgh = mg(0) = 0$

using equation of motion under gravity

$$v^2 - u^2 = 2gs$$

$$\Rightarrow v^2 - 0 = 2g(H-0)$$

$$\Rightarrow v = \sqrt{2gH}$$

Putting this value of v in equation of kinetic energy

we get, $K.E. = \frac{1}{2}mv^2$

$$= \frac{1}{2}m(\sqrt{2gH})^2$$

$$= \frac{1}{\cancel{2}}m\cancel{2}gH$$

$$= mgH$$

Total mechanical energy = K.E. + P.E.

$$= mgH + 0$$

$$= mgH$$

Since the total mechanical energy of the system remain constant for a freely falling body under gravity. so we can say the mechanical energy is conserved.

TRANSFORMATION OF ENERGY.

① Potential energy to kinetic energy.

Eg:- waterfall, skydiver

② kinetic energy to gravitational potential energy.

Eg:- satellites at a instant of time

③ Gravitational potential energy to electrical energy.

Eg:- Hydroelectric dam.

(4) Kinetic/mechanical energy to thermal/heat energy.

Eg - Rubbing both the hands.

(5) Heat energy to mechanical energy.

Eg ÷ steam engine.

(6) Heat energy to electrical energy

Eg ÷ thermocouple, thermo power plant.

(7) Electric energy to heat energy

Eg ÷ Room heater, water heater, oven.

(8) Mechanical energy to electric energy.

Eg ÷ Electric generator

(9) Electric energy to mechanical energy.

Eg ÷ Electric motor, blender, juicer-mixer, fan.

(10) Electrictric energy to chemical energy.

Eg ÷ Electroplating, charging ~~lithium ion~~ Li-ion battery

(11) chemical energy to electric energy.

Eg ÷ battery powered torchlight, fuel cell

(12) chemical energy to mechanical & electric energy.

Eg ÷ Movement of human body

(13) chemical energy to heat energy & radiant energy

Eg ÷ Burning of wood or coal

(14) Sound energy to electric energy.

Eg ÷ Microphone

(15) Electric energy to sound energy.

Eg ÷ loudspeaker, a sound amplifier.

(16) Electric energy to heat energy & radiant energy.

Eg:- An electric bulb.

(17) Elastic strain energy to electrical energy.

Eg:- in piezoelectric (gas lighter)

(18) wind energy to mechanical or electric energy.

Eg:- Windmills.

(19) solar energy to chemical energy.

Eg:- photosynthesis.

(20) solar energy to electric energy.

Eg:- solar cells or photovoltaic cells.

POWER - The rate of doing work is called power.

Mathematically, $\text{Power} = \frac{\text{Work}}{\text{Time}}$

$$\Rightarrow P = \frac{W}{T}$$

$$\Rightarrow P = \frac{FS}{T} \quad \because \{W = F.S\}$$

$$\Rightarrow P = F.V \quad \because \left\{\frac{S}{T} = V\right\}$$

$$\Rightarrow \text{Power} = \text{Force} \times \text{velocity}.$$

instaneous power.

$$P = \frac{dW}{dt}$$

$$\Rightarrow P = \frac{d(FS)}{dt}$$

$$\Rightarrow P = F\frac{ds}{dt}$$

$$\Rightarrow P = F.V$$

$\therefore$ Power = Force $\times$ velocity.

→ Power is a scalar quantity.

→ it si unit is watt $= \frac{1J}{1s}$

→ it dimensional formula → Power $= \frac{\text{work}}{\text{Time}}$

$$\rightarrow \text{Power} = \frac{[M^1L^2T^{-2}]}{[M^0L^0T^1]}$$

$$\rightarrow \text{Power} = [M^1L^2T^{-3}]$$

Proble-1

convert 24.5 Gev to Joules. 1 ev $= 1.6\times10^{-19}$ J

→ 1 Gev $= 1.6\times10^{-19} \times 10^{+9} = 1.6\times10^{-10}$

so, 24.5 Gev $= 24.5 \times 1.6 \times 10^{-10} = 39.2 \times 10^{-10}$ J Ans

Problem-2

55 kg & 52 kg athletes run the track of 200 meters in 19.19 sec & 24.34 sec respectively. who has more muscular power? consider $g = 9.8\ m/s^2$

→ Given

$M_1 = 55$ kg, $T_1 = 19.19$ sec, $S = 200$ m

$M_2 = 52$ kg, $T_2 = 24.34$, $S = 200$ m

$g = 9.8\ m/s^2$

so, the power$_1$ = F.V. (force x velocity)

$= mg \times \frac{S}{T}$ $\left\{ F = ma = mg \right.$ acceleration due to gravity

$V = S/T$

$= 55 \times 9.8 \times \frac{200}{19.19}$

$= 5617.51 W = 5.617$ K.W

Similarly power$_2$ = $mg \times \frac{S}{T}$

$= \cancel{52 \times 21.34}$

$= 52 \times 9.8 \times \frac{200}{21.34}$

$= 4776.00 \, W$

$= 4.776 \, kW$.

∴ The 55 kg athlete use more muscular power.

Problem-3

If a hydropower plant, nuclear power plant & thermal power plant generate maximum output of 4.3 GW, 850 MW, & 2100 MW at full load for 6 hours/day, 12 hrs/day, & 8 hrs/day respectively then calculate the energy produce in an hour for each plant.

⇒ For hydropower plant :-

Power = 4.3 GW = 4.3×10^3 = 4300 MW for 6 hrs/day.

⇒ $\frac{4300}{6}$ for 1 hrs/day.

⇒ 716.67 MW.

∴ $E = W = P.t = 716.67 \times 1$

$= 716.67$ MWh

For nuclear power plant :-

Power = 850 MW for 12 hrs/day.

⇒ $\frac{850}{12}$ for 1 hr/day

⇒ 70.83 MW

Energy produce = $w \times t = 70.83 \times 1 = 70.83$ MWh.

For thermal power plant :-

Power = 2100 MW for 8 hrs/day

⇒ $\frac{2100}{8}$ for 1 hrs/day

⟹ 262.5 MW for 1 hr

∴ Energy produce = W × T = 262.5 × 1 = 262.5 MWh

∴ The hydropower plant ~~pow~~ provide more energy in an hr

3.4 Annine uses 2 unit of power supply for domestic purpose per day & the electricity board charges 1.25 Rs per kWh, calculate the total energy spent per month corresponding ~~elecity~~ electricity bill amount.

3.5 A gold bar with a mass of 10 kg rests on a plane inclined at 45° from horizontal. Find out the component of the weight of the gold bar which is parallel to the inclined plane.

⟹ given

mass of gold = 10 kg.

the bar rest on inclined at 45° from horizontal.

so, the parallel component of weight = $mg \sin\theta$

$= 10 \times 9.8 \times (\sin 45°)$

$= 10 \times 9.8 \times \frac{1}{\sqrt{2}}$

$= \frac{98}{\sqrt{2}}$

$= \frac{98}{1.41}$

$= 69.503$ kg.

Answer :- 3.4

Given :- 2units power supply per day.

so the power supply for 1month or 30 days

$= 30 \times 2 = 60$ unit.

so, Electricity bill amount $= 60 \times 1.25 =$ ~~140~~ ~~Rs~~ 75 Rs