



**LECTURE NOTES
ON
Structural Design-I (Th.1)
4th Semester**

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1 Working stress method (WSM)

1.1 Objectives of design and detailing. State the different methods of design of concrete structures

1.2 Introduction to reinforced concrete, R.C. sections their behavior, grades of concrete and steel. Permissible stresses, assumption in W.S.M.

1.3 Flexural design and analysis of single reinforced sections from first principles.

1.4 Concept of under reinforced, over reinforced and balanced sections.

1.5 Advantages and disadvantages of WSM, reasons for its obsolescence.

WORKINGSTRESSMETHOD

Objectiveofdesignanddetailing:

Everystructuremustbe**designed**tosatisfythreebasic requirements;

- **Stability** to prevent overturning, sliding or buckling of the structure, or part of it, under the action of load.
- **Strength** to resist safely the stresses induced by the loads in the various structural members.
- **Serviceability** to ensure satisfactory performance under service load condition. Serviceability includes two parameters i.e deflection and cracking. The deflection should be limited to ensure the better appearance of the structure and to prevent cracking. The cracking of the reinforced concrete should not be excessive to ensure better appearance and also to prevent the access of water from cracks which may corrode the reinforcement.

There are two other considerations that a sensible designer ought to bear in mind, viz. **economy** and **aesthetics**.

Different methods of design:

(a) Working stress method:

- The earliest codified design philosophy is that of **working stress method** of design (WSM).
- Close to a hundred years old, this traditional method of design, based on linear elastic theory is still surviving in a number of countries.
- In WSM it is assumed that structural material e.g. concrete and steel behave in linearly elastic manner and adequate safety can be ensured by restricting the stresses in the material induced by working loads (service loads) on the structure.
- As the specified permissible (allowable) stresses are kept well below the material strength, the assumption of linear elastic behavior is considered justifiable.
- The ratio of the strength of the material to the permissible stress is often referred to as the **factor of safety**.

(b) Ultimate load method:

- In this method the inelastic behavior of concrete is taken into account and therefore reserve strength of concrete can be used leading to the economical design.
- In design, the loads on the structure are increased by suitable load factor and the structure is loaded with these increased loads called ultimate load.
- For material (steel and concrete) the ultimate behavior is taken into account.
- The ultimate load method leads the design to economy but at the same time, to very slender section, larger deflection and larger crack width in concrete.

(c) Limit state method:

- In this method the design based on limit state concept, the structure shall be designed to withstand safely all loads liable to act on it throughout its life.
- It shall also satisfy the serviceability requirement such as prevention of excessive deflection and cracking.
- WSM gives satisfactory performance of the structure at working loads and it is unrealistic at ultimate state of collapse.
- ULM provides realistic assessment of safety but it does not guarantee the satisfactory serviceability requirement at service load.
- The acceptable limit for the safety and serviceability requirement before failure occurs is called "limit state"
- The aim of the design is to achieve acceptable probabilities that the structure will not become unfit for the use for which it is intended, that means it will not reach a limit state. For ensuring the above objective, various partial factors of safety are employed in the limit state design.
- The design values are obtained from the characteristic values through the use of partial factor of safety. Design loads are obtained by multiplying a partial factor of safety for loads with characteristic load and in a similar manner the design strength of material are obtained by dividing the characteristic strength with respective partial factor of safety for materials.

Reinforced cement concrete:

- It is a combination of concrete and steel to build a structure instead of using only concrete.
- Concrete is good in resisting compression but is weak in resisting tension.
- On the other hand steel has high tensile strength and the bond between concrete and steel is good.
- To overcome the drawback of concrete, steel bars are used along with concrete.

Grades of concrete:

- The properties of concrete vary so much with composition and method of mixing therefore different types of concrete can be obtained.
- Concrete mixes have been classified into various grades by the Indian Standard Institution.

Types of concrete	Grade designation	Characteristic compressive strength at 28 days in N/mm ²
Normal strength concrete	M5	5
	M10	10
	M15	15
Standard concrete	M20	20
	M25	25
	M30	30
High strength concrete	M35 onwards	35 onwards

- In this designation M refers to mix and the number represents the characteristic compressive strength of cube at 28 days expressed in N/mm²
- Characteristic strength is defined as the strength of the material below which not more than 5% of the test results are expected to fall.

Advantages of reinforced cement concrete:

- Reinforced Cement Concrete has good compressive strength (because of concrete).
- RCC also has high tensile strength (because of steel).
- It has good resistance to damage by fire and weathering (because of concrete).
- RCC protects steel bars from buckling and twisting at the high temperature.
- RCC prevents steel from rusting.
- Reinforced Concrete is durable.
- The monolithic character of reinforced concrete gives it more rigidity.
- Maintenance cost of RCC is practically nil.

Concept of under, balanced and over reinforced beam section:**Under reinforced section:**

- In some cases a larger section is provided than required for a balanced section. In other words, a smaller area of steel is provided than required for a balanced section.
- In this case at some value of loads, the stress in steel will reach its permissible or design value and fail while concrete stress is less than its permissible value.
- The failure in this case is tension or ductile failure.

Balanced reinforced section:

- In this type of design, the section is so proportioned that the steel and concrete both reach their maximum values of stress at the same time. Thus at some value of load both the material will fail at the same time.

Overreinforced:

- In this case steel area provided is more than the area required for a balanced section.
- At some value of load the stress in concrete will reach its permissible value and fails while stress in steel is less than its permissible value.
- The failure in this case is called compression failure and therefore it will be a brittle failure.

Working stress method:**Assumptions:**

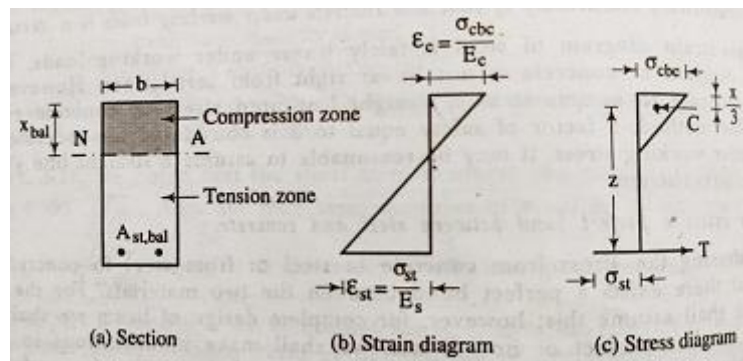
The analysis and design of a RCC member are based on the following assumptions

- Concrete is assumed to be homogeneous
- At any cross section, plane sections before bending remain plane after bending.
- The stress-strain relationship for concrete is a straight line, under working loads
- The stress-strain relationship for steel is a straight line, under working loads
- Concrete area on tension side is assumed to be ineffective
- All tensile stresses are taken up by reinforcements and none by concrete except when specially permitted.
- The steel area is assumed to be concentrated at the centroid of the steel.
- The modular ratio has the value $280/3\sigma_{cbc}$ where σ_{cbc} is permissible stress in compression due to bending in concrete in N/mm^2 as specified in code (IS:456-2000).

Permissible stresses:

- In working stress method, the stresses in materials are not exceeded beyond their permissible values. The permissible stresses are found by using suitable factor of safety to the material strength.
- For concrete in compression in bending a factor of safety equal to 3 is considered on characteristic strength of concrete.
- For steel a factor of safety equal to 1.8 is considered on yield strength of steel in tension due to bending

Derivation of formula for balanced design: consider a singly reinforced beam with stress and strain diagram as shown in figure.



$A_{st,bal}$ = reinforcement area provided for balanced section
 σ_{cbc} = permissible stress in concrete in bending compression
 σ_{st} = permissible stress in steel in tension
 E_c = modulus of elasticity of concrete
 E_s = modulus of elasticity of steel
 ϵ_c = strain in concrete in extreme compression fibre = σ_{cbc} / E_c
 ϵ_{st} = strain in steel = $\sigma_{st} / E_s = \sigma_{st} / m E_c = w_i$
 d = depth of beam
 d = effective depth which is defined as the distance from extreme compression fibre to the centroid of tensile reinforcement
 x = depth of N.A which is defined as the distance of neutral axis from extreme compression fibre.
 z = lever arm which is defined as the distance between centroid of compressive force to the centroid of tensile force

To find Neutral axis

: From the strain diagram

$$\frac{x_{bal}}{d - x_{bal}} = \frac{\sigma_{cbc} / E_c}{\sigma_{st} / E_s} = \frac{m \sigma_{cbc}}{\sigma_{st}}$$

$$X_{bal} = \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} d = \frac{1}{1 + \frac{\sigma_{st}}{m \sigma_{cbc}}} d$$

$$X_{bal} = kd$$

$$\text{Where } k = \frac{1}{1 + \frac{\sigma_{st}}{m \sigma_{cbc}}}$$

and is known as neutral axis constant

To find lever arm:

From stress

$$\text{diagram } z = d - \frac{x_{bal}}{3}$$

$$= d - \frac{kd}{3} = \left(1 - \frac{k}{3}\right) d$$

$$= jd$$

Where the constant $j = \left(1 - \frac{k}{3}\right)$

Andisknownasleverarmconstant

To find total forces:

$C = \text{total compression}$

$T = \text{total tension}$

$C = \frac{1}{2} \sigma_{cbc} b x_{bal}$

$$= \frac{b x_{bal} \sigma_{cbc}}{2}$$

$T = \sigma_{st} A_{st, bal}$

To find moment of resistance of section

Capacity of a section to resist the moment is known as its moment of resistance.

$M.R = \text{total compressive force} \times \text{lever}$

arm Or,

$M.R = \text{total tensile force} \times \text{lever}$

arm Considering compressive force

$M.R = \text{total compression} \times \text{lever arm}$

$= (\frac{1}{2} \sigma_{cbc} b x_{bal}) j d$

$= (\frac{1}{2} \sigma_{cbc} b k d) j d$

$=$

$(\frac{1}{2} \sigma_{cbc} k j) b d^2$

$= Q_{bal} b d^2$

Where $Q_{bal} = \frac{1}{2} \sigma_{cbc} k j$ and is known as moment of resistance

factor Considering the tensile forces

$M.R = \text{total tension} \times \text{lever arm}$

$= A_{st, bal} \sigma_{st} j d$

steel area $= A_{st} = A_{st, bal} = \frac{M}{\sigma_{st} j d}$

Question1

Calculate the design constants for the following material considering the balanced design for or singly reinforced section. The materials are M20 grade concrete and mild steel reinforcement.

Solution:

For M20 grade concrete, $\sigma_{cbc} =$

7 N/mm^2 For Fe250 grade steel, $\sigma_{st} = 140 \text{ N/mm}^2$

$$\text{Modular ratio} = m = \frac{280 - 280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$\text{Neutral axis constant} = k = \frac{1}{1 + \frac{\sigma_{st}}{m\sigma_{cbc}}} = \frac{1}{1 + \frac{140}{13.33 \times 7}} = 0.4$$

$$\text{Lever arm constant} = j = (1 - k) = \frac{1 - 0.4}{3} = 0.87$$

$$\text{M.R constant} = Q_{bal} = \frac{1}{2} \sigma_{cbc} k j = \frac{1}{2} \times 7 \times 0.4 \times 0.87 = 1.21$$

$$P_{tbal} = \frac{50 \sigma_{cbc} \times k}{\sigma_{st}} = \frac{50 \times 7 \times 0.4}{140} = 1.0$$

Question2 :

For a rectangular beam of size 250 mm wide x 520 mm effective depth, find out the balanced depth of neutral axis, balanced lever arm, balanced moment of resistance and balanced steel area. The materials are M20 grade concrete and HYSD reinforcement of grade Fe415.

Solution:

$$b = 250$$

$$m = 520 \text{ mm}$$

$\sigma_{cbc} = 7$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$k = 0.29$$

$$j = 0.90$$

$$Q_{bal} = 0.91$$

$$\text{Depth of}$$

neutral axis

$$k d = 0.29 \times 520 = 150.8 \text{ mm}$$

$$\text{Lever arm} = j d = 0.9 \times 520 = 468 \text{ mm}$$

$$\text{M.R of balanced section} = M = Q_{bal} b d^2 = 0.91 \times 250 \times 520 \times 520 \times 10^{-6} = 61.52 \text{ KNm}$$

$$A_{stbal} = \frac{P_{tbal} \times b d}{100} = \frac{1.0 \times 250 \times 520}{100} = 520 \text{ mm}^2$$

Question3

A simply supported rectangular beam of 4m span carries a uniformly distributed load of 26 KN/m. The width of the beam is 230 mm. find the depth and steel area for balanced design. Use M20 grade concrete and mild steel reinforcement

Solution:

Maximum moment $M = 26 \times 4^2 / 8$
 $= 52 \text{ Kn.m}$

For balanced section $Q_{bal} = 1.21$

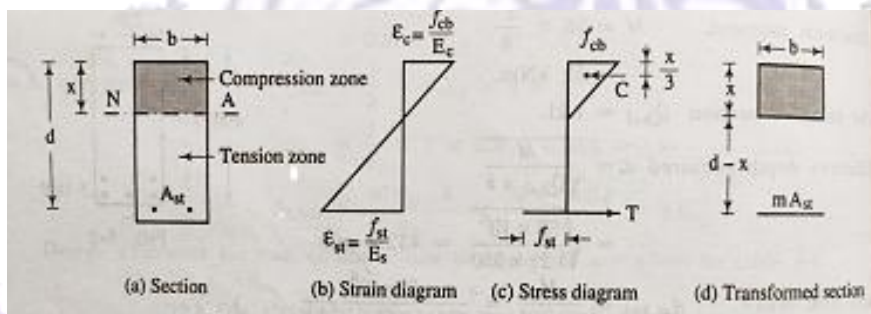
Effective depth required $= d = \frac{\sqrt{M_{bal}}}{b Q_{bal}} = \frac{\sqrt{52 \times 10^6}}{1.21 \times 230} = 432.3 \text{ mm}$

Steel area $= A_{stbal} = \frac{M}{\sigma_{stj} d} = \frac{52 \times 10^6}{140 \times 0.87 \times 432.3} = 988 \text{ mm}^2$

Provide 5 nos. of 16 mm diameter bars giving area of $5 \times 201 = 1005 \text{ mm}^2$

Transformed area method:

- A transformed section is a section in which the steel area is replaced by an equivalent concrete area.
- A transformed section consists of a single material, therefore, theory of simple bending can be applied.
- The transformed section may be of steel when concrete is replaced by steel or it may be of concrete when steel area is replaced by concrete. It is usual to replace steel area by concrete, hence a transformed section would mean to a homogenous concrete section.
- The actual concrete in tension zone is absent because we have assumed that concrete carry tensile force. Thus all tensile forces will be carried by steel.



At the centroid of steel reinforcement, the surrounding concrete being elastic and having perfect bond with steel

Strain in steel = strain in concrete

Let f_{st} and f_{cb} be the stresses in steel and concrete respectively at the level of centroid of steel. Strain in concrete = strain in steel

$$\frac{f_{cb}}{E_c} = \frac{f_{st}}{E_s} = m f_{cb}'$$

Now force in steel $= A_{st} f_{st} = A_{st} m f_{cb}'$ ----- (1)

If this steel is to be replaced by an equivalent concrete area, the equivalent concrete will carry the same force

Now the force in equivalent concrete = transformed area $\times f_{cb}'$ ----- (2)

Equating(1)&(2)

Transformed area $f_{cb}' = A_{st} m f_{cb}$

transformed area $= m A_{st}$

To find neutral

axis: $b \times x \times (x/2) = m A_{st} ($

$d-x)$ lever arm $= d - (x/3)$

stress in steel $f_{st} = \frac{M}{(d - \frac{x}{3}) A_{st}}$

Stress in concrete:

From strain diagram, $\frac{f_{cb}}{E_c} = \frac{f_{st}}{E_s} \frac{x}{d-x}$

$f_{cb} = \frac{f_{st}}{E_s} \times \frac{x}{d-x} \times \frac{E_c}{m}$

M. R in compression $= \sigma_{cbc} (bx/2) (L.A)$

M. R in tension $= \sigma_{st} A_{st} (L.A)$

Question 4:

A rectangular beam of width 200 mm and effective depth 460 mm reinforced with 3-16mm diabar. The section is subjected to a characteristic moment of 30 KNm. Determine the maximum stress in steel and concrete. The materials are M20 grade concrete and mild steel reinforcement. Also find out the M. R of this section.

Solution:

For M20 grade concrete and Fe 250 m

$= 13.33$

transformed area of steel $= 13.33 \times 603$

$= 8038 \text{ mm}^2$

To find neutral axis, taking moments of transformed area about N.A $200 \times x \times (x/$

$2) = 8038(460-x)$

$100x^2 + 8038x - 36987480 = 0$

$x = 156.2 \text{ mm}$

lever arm $= 460 - (156.2/3) = 407.9 \text{ mm}$

steel stress $= \frac{30 \times 10^6}{603 \times 407.9} = 121.97 \text{ N/mm}^2$

concrete stress $= \frac{f_{st}}{m} \times \frac{x}{d-x} = \frac{121.97}{13.33} \times \frac{156.2}{303.8} = 4.7 \text{ N/mm}^2$

M. R in compression $= \frac{7 \times 200 \times 156.2 \times 407.9 \times 10^{-6}}{2} = 44.6 \text{ KNm}$

M. R in tension $= 140 \times 603 \times 407.9 \times 10^{-6} = 34.43 \text{ KNm}$

M. R of this section $= 34.43 \text{ KNm}$

Analysis of this section:

Type 1: to find out the depth of neutral axis for a given section and specifying the type of beam

- If the section and steel area are given, find out neutral axis by taking moment of transformed areas about Neutral Axis.

$$b \times x \times (x/2) = m A_{st} (d - x)$$

- Find out

depth of neutral axis for a balanced section. $x = kd$

$$\text{where } k = \frac{1}{1 + m \rho_{bc}}$$

- If $x_{act} < x_{bal}$, -
the beam section is underreinforced. If $x_{act} > x_{bal}$, -
the beam section is overreinforced
If $x_{act} = x_{bal}$, - the beam section is balanced

Type 2: to find the moment of resistance for a given section

- Find the position of actual and balanced N.A. as explained above.
- If $x_{act} < x_{bal}$, the beam section is underreinforced and moment of resistance is given by
 $M.R = A_{st} \sigma_{st} (d - x/3)$
- If $x_{act} > x_{bal}$, the beam section is overreinforced and moment of resistance is given by $M.R = b \times x \times (\sigma_{cbc}/2) (d - x/3)$

Design of the section :

Type-1: dimension not given

- The moment of resistance of balanced section $M_{bal} = Q_{bal} b d^2$. Out of the two variables "b" and "d", one must be known to us. It is usual to fix the width (b) of the section.
- Once width is fixed, the depth will be calculated from the following formula

$$d = \sqrt{\frac{M}{Q_{bal} \times b}}$$

here $M = M_{bal}$, where M is the applied moment.

- The area of steel (A_{st}) will be calculated as per the following formula

$$A_{st} = \frac{M}{\sigma_{st} \times j d}$$

Type 2: dimensions are given

- Applied moment M, the section dimensions b and d are given
- Determine $M_{bal} = Q_{bal} b d^2$
- If $M < M_{bal}$, the section is designed as underreinforced beam.
- If $M > M_{bal}$, the section is designed as overreinforced beam.
- If $M = M_{bal}$, the section is designed as balanced.

Question 5:

An R.C.C beam, 300 mm wide and 460 mm effective depth is reinforced with 4 nos. of 12 mm dia bars in tension. Find out the depth of neutral axis and type of the beam. The materials are M20 grade concrete and Fe415 grade steel.

Solution:

For M20 grade concrete, $m = 13.33$

$$A_{st} = 4 \times 113 = 452 \text{ mm}^2$$

Let x be the depth of N.A

Taking moment of transformed areas about

$$N.A \times x \times (x/2) = m A_{st} (d - x)$$

$$(300/2)x^2 = 13.33 \times 452 (460 -$$

$$x)x^2 + 40.17x - 18477 = 0$$

$$x = 117.3 \text{ mm}$$

$$\text{depth of balanced N.A} = 0.29 \times 460 = 133.4$$

$$\text{mm } x_{\text{actual}} < x_{\text{bal}}$$

The beam is underreinforced.

Question 6:

Find the M.R of a beam having width 230 mm and 560 mm effective depth reinforced with 3 nos. of 20 mm dia bar. Also state the type of the beam. The materials are M20 grade concrete and Fe415 grade steel.

Solution:

For M20 grade concrete, $m =$

$$13.33 A_{st} = 3 \times 314 = 942 \text{ mm}^2$$

Let x be the depth of N.A

Taking moment of transformed areas about

$$N.A \times x \times (x/2) = m A_{st} (d - x)$$

$$(230/2)x^2 = 13.33 \times 942 (560 -$$

$$x)x^2 + 109.2x - 61146 = 0$$

$$x = 198.6 \text{ mm}$$

$$\text{depth of balanced N.A} = kd = 0.29 \times 560 = 162.4$$

$$\text{mm } x_{\text{actual}} > x_{\text{bal}}$$

The beam is overreinforced and concrete will fail first.

$$M.R = b \times x \times (\sigma_{bc} / 2) (d - x/3)$$

$$= 230 \times 198.6 \times (7/2) (560 - 198.6/3) \times 10^{-6}$$

$$= 78.95 \text{ Knm}$$

Question 7:

Design a reinforced concrete beam subjected to a bending moment of 20 Knm. Use M20 Grade concrete and Fe 415 grade steel. Keep the width of the beam equal to half the effective depth.

Solution:

For M20 grade concrete, $m = 13.33$

$$\text{Neutral axis constant } = k = \frac{1}{1 + \frac{\sigma_{st}}{m \sigma_{bc}}} = \frac{1}{1 + \frac{230}{13.33 \times 7}} = 0.29$$

$$\text{Lever arm constant} = j = (1 - k) = \frac{(1 - 0.29)}{3} = 0.90$$

$$M_{\text{bal}} = Q_{\text{bal}} b d^2$$

$$= \frac{1}{2} \sigma_{\text{cbc}} k j b d^2 = \frac{1}{2} \times 7 \times 0.29 \times 0.90 \times (d/2) \times d$$

$$= 0.457 d^3$$

$$M = 20 \text{ Knm} = 20 \times 10^6 \text{ Nmm}$$

$$M_{\text{bal}} = M$$

$$0.457 d^3 = 20 \times 10^6$$

$$d = 353 \text{ mm}$$

$$b = 353/2 = 177 \text{ mm}$$

$$\text{steel area} = A_{\text{st}} = A_{\text{stbal}} = \frac{M}{\sigma_{\text{st}} j d}$$

$$A_{\text{st}} = \frac{20 \times 10^6}{230 \times 0.90 \times 353} = 273 \text{ mm}^2$$



2 Philosophy Of Limit State Method (LSM)

2.1 Definition, Advantages of LSM over WSM, IS code suggestions regarding design philosophy.

2.2 Types of limit states, partial safety factors for materials strength, characteristic strength, characteristic load, design load, loading on structure as per I.S. 875

2.3 Study of I.S specification regarding spacing of reinforcement in slab, cover to reinforcement in slab, beam column & footing, minimum reinforcement in slab, beam & column, lapping, anchorage, effective span for beam & slab.

PHILOSOPHY OF LIMIT STATE METHOD **SAFETY AND SERVICEABILITY REQUIREMENTS**

In the method of design based on limit state concept, the structure shall be designed to withstand safely all loads liable to act on it throughout its life; it shall also satisfy the serviceability requirements, such as limitations on deflection and cracking. The acceptable limit for the safety and serviceability requirements before failure occurs is called a 'limit state'. The aim of design is to achieve acceptable probabilities that the structure will not become unfit for the use for which it is intended that it will not reach a limit state.

All relevant limit states shall be considered in design to ensure an adequate degree of safety and serviceability. In general, the structure shall be designed on the basis of the most critical limit state and shall be checked for other limit states.

For ensuring the above objective, the design should be based on characteristic values for material strengths and applied loads, which take into account the variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data if available; where such data are not available they should be based on experience. The 'design values' are derived from the characteristic values through the use of partial safety factors, one for material strengths and the other for loads. In the absence of special considerations these factors should have the values given in 36 according to the material, the type of loading and the limit state being considered.

Limit State of Collapse

The limit state of collapse of the structure or part of the structure could be assessed from rupture of one or more critical sections and from buckling due to elastic or plastic instability (including the effects of sway where appropriate) or overturning. The resistance to bending, shear, torsion and axial loads at every section shall not be less than the appropriate value at that section produced by the probable most unfavourable combination of loads on the structure using the appropriate partial safety factors.

Limit State Design

For ensuring the design objectives, the design should be based on characteristic values for material strengths and applied loads (actions), which take into account the probability of variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data, if available. Where such data is not available, they should be based on experience. The design values are derived from the characteristic values through the use of partial safety factors, both for material strengths and for loads. In the absence of special considerations, these factors should have the values given in this section according to the material, the type of load and the limit state being considered. The reliability of design is ensured by requiring that

$$\text{Design Action} \leq \text{Design Strength.}$$

Limit

Limit states are the states beyond which the structure no longer satisfies the performance requirements specified. The limit states are classified as

- a) Limit state of strength
- b) Limit state of serviceability

a) The limit state of strength are those associated with failures (or imminent failure), under the action of probable and most unfavorable combination of loads on the structure using the appropriate partial safety factors, which may endanger the safety of life and property. The limit state of strength includes:

- Loss of equilibrium of the structure as a whole or any of its parts or components.
- Loss of stability of the structure (including the effect of sway where appropriate and overturning) or any of its parts including supports and foundations.
- Failure by excessive deformation, rupture of the structure or any of its parts or components.
- Fracture due to fatigue.
- Brittle fracture.

b) The limit state of serviceability include

- Deformation and deflections, which may adversely affect the appearance or, effective, use of the structure or may cause improper functioning of equipment or services or may cause damage to finishes and non-structural members.
- Vibrations in the structure or any of its components causing discomfort to people, damage to the structure, its contents or which may limit its functional effectiveness. Special consideration shall be given to floor vibration systems susceptible to vibration, such as large open floor areas free of partitions to ensure that such vibrations is acceptable for the intended use and occupancy.
- Repairable damage due to fatigue.
- Corrosion and durability.

Limit States of Serviceability

To satisfy the limit state of serviceability, the deflection and cracking in the structures shall not be excessive. This limit state corresponds to deflection and cracking.

Deflection

The deflection of a structure or part shall not adversely affect the appearance or efficiency of the structure or finishes or partitions.

Cracking

Cracking of concrete should not adversely affect the appearance or durability of the structure; the acceptable limits of cracking would vary with the type of structure and environment. The actual width of cracks will vary between the wide limits and predictions of absolute maximum width are not possible. The surface width of cracks should not exceed 0.3 mm.

In members where cracking in the tensile zone is harmful either because they are exposed to the effects of the weather or continuously exposed to moisture or in contact with soil or ground water, an upper limit of 0.2 mm is suggested for the maximum width of cracks. For particularly aggressive environment, such as the 'severe' category, the assessed surface width of cracks should not in general, exceed 0.1 mm.

CHARACTERISTIC AND DESIGN VALUES AND PARTIAL SAFETY FACTORS

1. Characteristic Strength of Materials

Characteristic strength means that value of the strength of the material below which not more than 5 percent of the test results are expected to fall and is denoted by f . The characteristic strength of concrete (f_{ck}) is as per the mix of concrete. The characteristic strength of steel (f_y) is the minimum stress or 0.2 percent of proof stress.

2. Characteristic Loads

Characteristic load means that value of load which has a 95 percent probability of not being exceeded during the life of the structure. Since data are not available to express loads in statistical terms, for the purpose of this standard, dead loads given in IS 875 (Part 1), imposed loads given in IS 875 (Part 2), wind loads given in IS 875 (Part 3), snow load as given in IS 875 (Part 4) and seismic forces given in IS 1893-2002 (part-I) shall be assumed as the characteristic loads.

Design values:

Material:

The design strength of material f_d is given by

$$F_d = \frac{f}{\gamma_m}$$

where f = characteristic strength of materials

γ_m = partial safety factor appropriate to the material

Loads:

The design load F_d is given by $F_d =$

$F \gamma_f$

Where F = characteristic load

γ_f = partial safety factor appropriate to the nature of loading

Partial Safety Factors:

1. Partial Safety Factor γ_f for Loads

Sr.No.	Load Combination	Ultimate Limit State	Serviceability Limit State
1	DL+LL	1.5(DL+LL)	DL+LL
2	DL+WL i) DL contributes to stability ii) DL assists overturning	0.9DL+1.5WL 1.5(DL+WL)	DL + WL DL+ WL
3	DL+LL+WL	1.2(DL+LL+WL)	DL+0.8LL+0.8WL

2. Partial Safety Factor γ_m for Material Strength

Sr.No.	Material	Ultimate Limit State	Serviceability Limit State
1	Concrete	1.50	$E_c = 5000 f_{ck}$ MPa
2	Steel	1.15	$E_s = 2 \times 10^5$ MPa

When assessing the strength of a structure or structural member for the limit state of collapse, the values of partial safety factor, should be taken as 1.5 for concrete and 1.15 for steel.

3 Analysis and Design of Single and Double Reinforced Sections (LSM)

3.1 Limit state of collapse (flexure), Assumptions, Stress-Strain relationship for concrete and steel, neutral axis, stress block diagram and strain diagram for singly reinforced section.

3.2 Concept of under- reinforced, over-reinforced and limiting section, neutral axis coefficient, limiting value of moment of resistance and limiting percentage of steel required for limiting singly R.C. section.

3.3 Analysis and design: determination of design constants, moment of resistance and area of steel for rectangular sections.

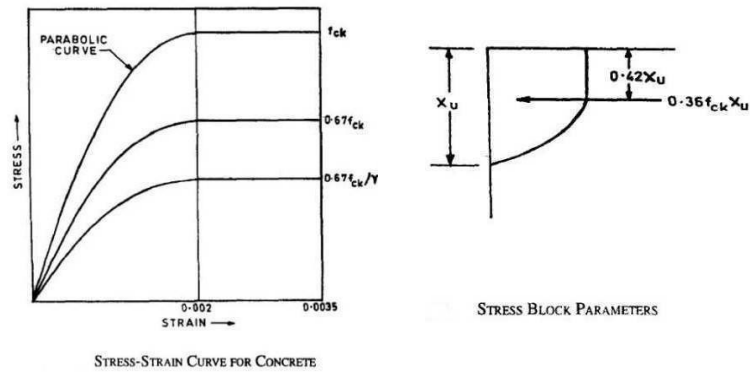
3.4 Necessity of doubly reinforced section, design of doubly reinforced rectangular section.

ANALYSIS AND DESIGN OF SINGLE AND DOUBLE REINFORCED SECTION

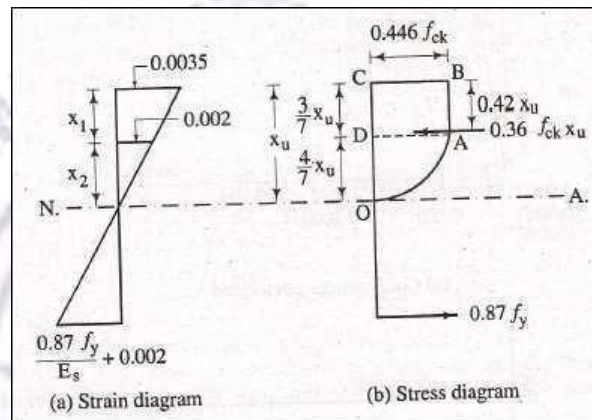
LIMIT STATE OF COLLAPSE: FLEXURE

Assumptions for Limit State of Collapse (Flexure):

- 1) Plane section normal to the axis remains plane even after bending, i.e. strain at any point on the cross section is directly proportional to the distance from the N.A.
- 2) Maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.
- 3) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is shown below



For design purposes, the compressive strength of concrete in the structures shall be assumed to be 0.67 times the characteristic strength. The partial safety factor $\gamma_m = 1.5$ shall be applied in addition to this.



NOTE-

For the above stress-strain curve the design stress block parameters are as follows:

$$\text{Area of parabolic portion} = \left(\frac{2}{3}\right) \times 0.446 f_{ck} \times \left(\frac{4}{7}\right) x_u$$

$$=$$

$$0.17 f_{ck} x_u$$

$$\text{Area of rectangular portion} = 0.446 f_{ck} \left(\frac{3}{7}\right) x_u$$

$$= 0.19 f_{ck} x_u$$

$$\text{Total area of stress block} = 0.17 f_{ck} x_u + 0.19 f_{ck} x_u = 0.36 f_{ck} x_u$$

Let \bar{y} be the distance of centroid of stress block from the extreme compression fibre

$$\bar{y} = \frac{0.17 f_{ck} x_u \left(x_1 + \frac{3}{8} x_2\right) + 0.19 f_{ck} x_u \left(\frac{x_1}{2}\right)}{0.36 f_{ck} x_u}$$

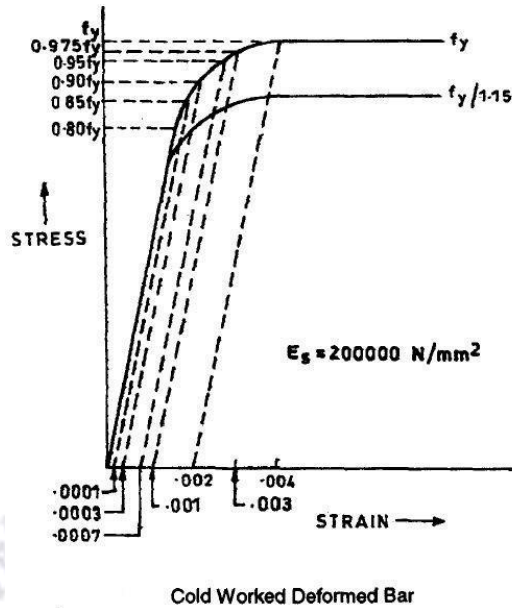
$$\text{substituting } x_1 = \left(\frac{3}{7}\right) x_u$$

$$x_2 = \left(\frac{4}{7}\right) x_u$$

Depth of centre of compressive force = $\bar{y} = 0.42 x_u$ from the extreme fibre in compression

Where f_{ck} = characteristic compressive strength of concrete, and x_u = depth of neutral axis

- 4) the tensile strength of the concrete is ignored.
- 5) the stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used.



- 6) the maximum strain in tension reinforcement in the section at failure shall not be less than
$$= \frac{0.87f_y}{E_s} + 0.002$$

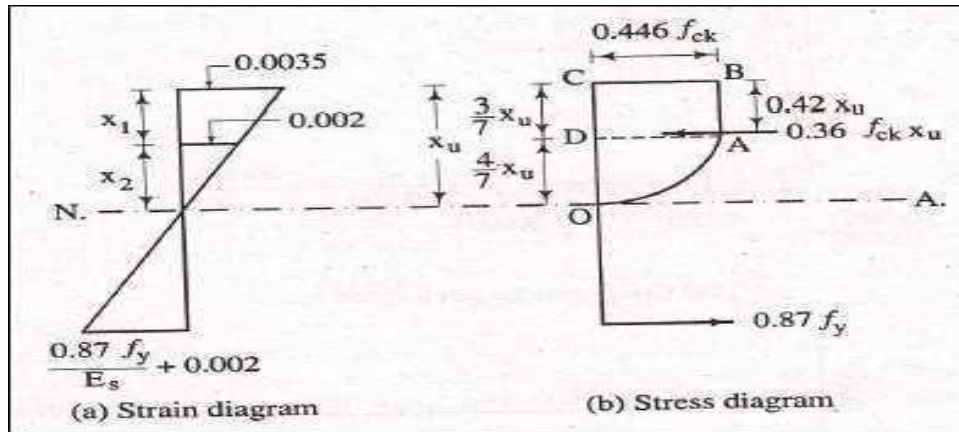
TYPES OF BEAM SECTIONS

- Section in which, tension steel reaches yield strain simultaneously as the concrete reaches the failure strain in bending are called, '**Balanced Section**'.
- Section in which, tension steel reaches yield strain at loads lower than the load at which concrete reaches the failure strain in bending are called, '**Under Reinforced Section**'
- Section in which, tension steel reaches yield strain at load higher than the load at which concrete reaches the failure strain in bending are called, '**Over Reinforced Section**'.

Analysis of singly reinforced rectangular beam

:

Derivation of formula:



A singly reinforced rectangular beam section with strain diagram and stress diagram are shown in figure.

To find Neutral axis:

Total compression = total

tension $0.36 f_{ck} b x_u = 0.87 f_y A_{st}$

$$X_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

- If $x_u < x_{u,max}$, this section is under reinforced section.
- If $x_u > x_{u,max}$, this section is over reinforced section, then $x_u = x_{u,max}$
- If $x_u = x_{u,max}$, this section is balanced section

Grade of steel	$x_{u,max}/d$
Fe250	0.53
Fe415	0.48
Fe500	0.46

To find lever arm:

From the stress diagram, the lever

$$\text{arm } Z = d - 0.42 x_u$$

To find moment of resistance:

(1) For a balanced section

$$\begin{aligned} M.R &= \text{total compression} \times \text{lever arm} \\ &= \text{total tension} \times \text{lever} \end{aligned}$$

$$\text{arm } M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

For limiting values substitute $x_{u\max}$ for x_u and $M_{u\lim}$ for M_u

$$\begin{aligned} M_{u\lim} &= 0.36 (x_{u\max}/d) (1 - 0.42 x_{u\max}/d) f_{ck} b d^2 \\ &= Q_{\lim} b d^2 \end{aligned}$$

Q_{\lim} = limiting moment of resistance factor for balanced rectangular section.

For under reinforced section

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

Types of problems:

Three different types of problems are considered for singly reinforced rectangular beams

Type 1:

To find out the depth of neutral axis and specifying the type of the beam

- For a given section, equate total tension and total compression and thus find out the depth of neutral axis using

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

- Also find out the limiting value of depth of neutral axis $x_{u\max}$, using $x_{u\max}/d$

- Then if

If $x_u < x_{u\max}$, the section is under reinforced section

If $x_u > x_{u\max}$, the section is over reinforced section, then $x_u = x_{u\max}$

If $x_u = x_{u\max}$, the section is balanced section

Type 2:

To find out moment of resistance of a given section

- Find out depth of neutral axis and type of the beam as discussed in type 1
- For over reinforced and balanced section, obtain moment of resistance by using the following equation

$$M_{u\lim} = 0.36 (x_{u\max}/d) (1 - 0.42 x_{u\max}/d) f_{ck} b d^2$$

- For under reinforced section obtain moment of resistance by using the following equation $M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

Or

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

Type

3: To design singly reinforced rectangular section for given width and applied factored moment

The width is usually decided by the functional or architectural requirement.

$$d = \sqrt{\frac{M}{Q_{lim} \times b}}$$

The steel area can be obtained by using the following formula

$$A_{st} = \frac{M_u}{0.87 f_y (d - 0.42 x_{u_{max}})}$$

Question 1:

A rectangular beam 230 mm wide and 520 mm effective depth is reinforced with 4 nos. of 16mm diameter bars. Find out the depth of neutral axis and specify the type of beam. The materials are M20 grade concrete and Fe415 grade steel. Also find out the depth of neutral axis if the reinforcement is increased to 4 nos. of 20mm dia bars.

Solution:

Case 1:

$$A_{st} = 4 \times 201 = 804 \text{ mm}^2$$

$$\text{Total compression} = 0.36 f_{ck} b x_u = 0.36 \times 20 \times 230 x_u = 1656 x_u \text{ Total}$$

$$\text{Total tension} = 0.87 f_y A_{st} = 0.87 \times 415 \times 804 =$$

$$290284 \text{ Equating total compression} = \text{total tension}$$

$$1656 x_u = 290284$$

$$x_u = 175.3 \text{ mm}$$

$$\text{limiting value of neutral axis} = x_{u_{max}} = 0.48 d = 0.48 \times 520 = 250$$

mm here $x_u < x_{u_{max}}$, this section is under reinforced section

Case 2:

$$x_u = 175.3 \text{ mm}$$

$$A_{st} = 4 \times 314 = 1256 \text{ mm}^2$$

$$\text{Total compression} = 0.36 f_{ck} b x_u = 0.36 \times 20 \times 230 x_u = 1656 x_u \text{ Total}$$

$$\text{Total tension} = 0.87 f_y A_{st} = 0.87 \times 415 \times 1256 = 453479$$

$$1656 x_u = 453479$$

$$x_u = 273.8 \text{ mm}$$

here $x_u > x_{u_{max}}$, this section is

over reinforced section $x_u = x_{u_{max}} = 250 \text{ mm}$

Question2:

A rectangular beam 230 mm wide and 460 mm effective depth is reinforced with 3 nos. of 20mm diameter bars. Find out the factored moment of resistance of the beam. The materials are M20 grade concrete and Fe415 grade steel. Also find out the factored moment of resistance if the reinforcement is increased to 5 nos. of 20mm dia bars.

Solution:

$$A_{st} = 3 \times 314 = 942 \text{ mm}^2$$

$$\text{Total compression} = 0.36 f_{ck} b x_u = 0.36 \times 20 \times 230 x_u = 1656 x_u \text{ Total}$$

$$\text{Total tension} = 0.87 f_y A_{st} = 0.87 \times 415 \times 942 =$$

$$340109 \text{ Equating total compression} = \text{total tension}$$

$$1656 x_u = 340109$$

$$x_u = 205.4 \text{ mm}$$

$$\text{limiting value of neutral axis} = x_{u\max} = 0.48d = 0.48 \times 460 =$$

$$220.8 \text{ mm where } x_u < x_{u\max}, \text{ the section is under reinforced section}$$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 230 \times 205.4 (460 - 0.42 \times 205.4) = 127.12 \times 10^6 = 127 \text{ Knm}$$

Case2:

$$A_{st} = 4 \times 314 = 1256 \text{ mm}^2$$

$$\text{Total compression} = 0.36 f_{ck} b x_u = 0.36 \times 20 \times 230 x_u = 1656 x_u \text{ Total}$$

$$\text{Total tension} = 0.87 f_y A_{st} = 0.87 \times 415 \times 1256 = 453479$$

$$1656 x_u = 453479$$

$$x_u = 273.8 \text{ mm}$$

$$x_{u\max} = 0.48d = 0.48 \times 460 = 220.8 \text{ mm}$$

here $x_u > x_{u\max}$, the section is

over reinforced section $x_u = x_{u\max} = 220.8 \text{ mm}$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 230 \times 220.8 (460 - 0.42 \times 220.8) = 134.2 \times 10^6 = 134 \text{ Knm}$$

Question3:

Design a singly reinforced rectangular beam for an applied factored moment of 120 Knm. Assume the width of section as 230mm. The materials are M20 grade concrete and Fe415 grade steel.

Solution:

$$M_u = 120$$

$$b = 230 \text{ mm}$$

m

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

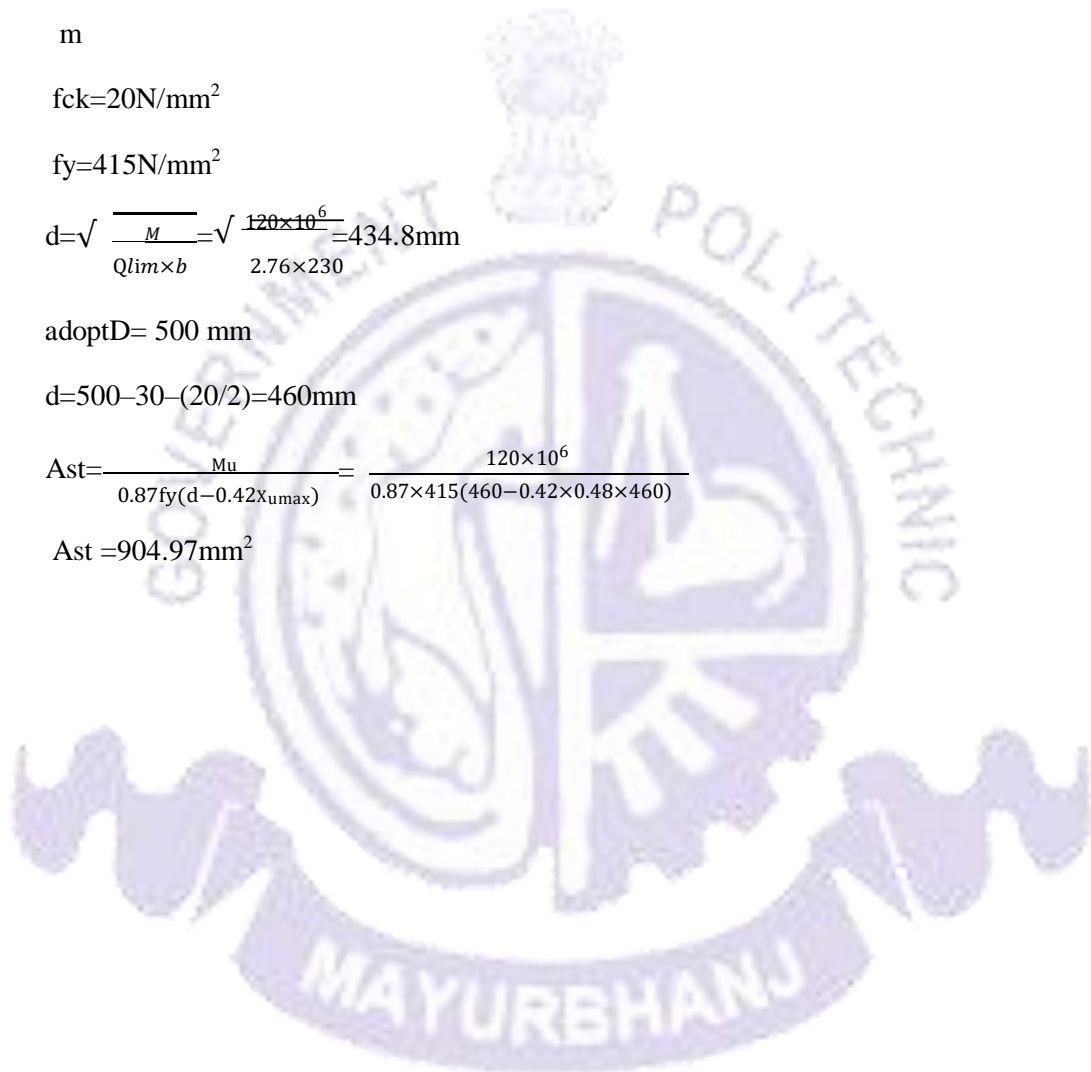
$$d = \sqrt{\frac{M_u}{Q_{lim} \times b}} = \sqrt{\frac{120 \times 10^6}{2.76 \times 230}} = 434.8 \text{ mm}$$

$$\text{adopt } D = 500 \text{ mm}$$

$$d = 500 - 30 - (20/2) = 460 \text{ mm}$$

$$A_{st} = \frac{M_u}{0.87 f_y (d - 0.42 x_{u_{max}})} = \frac{120 \times 10^6}{0.87 \times 415 (460 - 0.42 \times 0.48 \times 460)}$$

$$A_{st} = 904.97 \text{ mm}^2$$

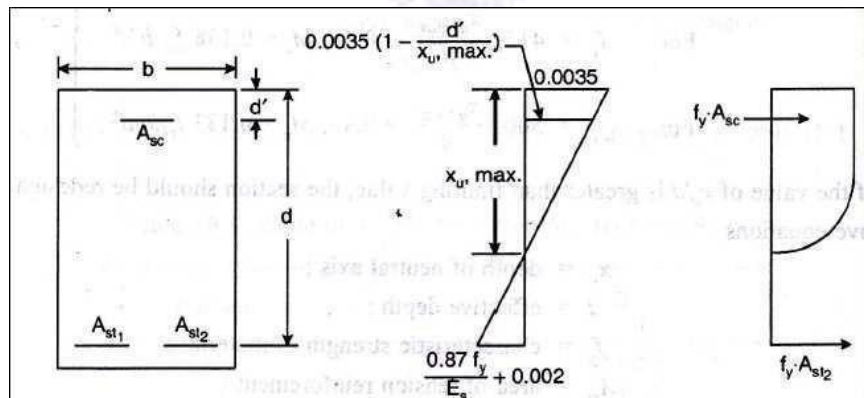


Doubly reinforced beam:

If the applied moment is greater than the M.R. of a singly reinforced section, there can be three Alternatives

- If possible increase the dimension of the section preferably depth
- Higher grade concrete can be used to increase the M.R. of the section
- Steel reinforcement may be added in compression zone to increase the M.R. of the section. This is known as doubly reinforced section.

Derivation of the formula



- A doubly reinforced beam section, strain diagram and stress diagram are shown in fig.
- A doubly reinforced beam subjected to a moment M_u can be expressed as a rectangular section with tension reinforcement A_{stlim} reinforced for balanced condition giving moment of resistance M_{ulim} + an auxiliary section reinforced with compression reinforcement A_{sc} and tensile reinforcement A_{st2} giving moment of resistance M_{u2} such that

$$M_u = M_{ulim} + M_{u2}$$

- For the moment M_{ulim} the tension steel A_{stlim} is found out as explained for singly reinforced beams.
- For the additional moment M_{u2} , the additional tension steel and compression steel are provided such that they give a couple of moment M_{u2}
- Let the compression reinforcement be provided at a depth d' from the extreme compression fibre. Then the lever arm for additional moment will be $(d - d')$
- Considering tension steel $M_{u2} = 0.87 f_y A_{st2} (d - d')$

Considering compression

$$M_{u2} = A_{sc} (f_{sc} - f_{cc}) (d - d')$$

Where, A_{st2} = Area of additional tensile

reinforcement A_{sc} = Area of compression reinfor

cement

f_{sc} = Stress in compression reinforcement

f_{cc} = Compressive stress in concrete at the level of

compression reinforcement

Since the additional reinforcement is balanced by the additional compressive force.

$$A_{sc} \cdot (f_{sc} - f_{cc}) = 0.87 f_y \cdot A_{st2}$$

$$A_{st2} = \frac{(f_{sc} - f_{cc})}{0.87 f_y}$$

Total area of reinforcement shall be obtained

$$\text{by } A_{st} = A_{st1} + A_{st2}$$

A_{st1} = Area of reinforcement for singly reinforced section for $M_{u,lim}$

The value of f_{sc} in N/mm^2 can be obtained from the following table for different values of d/d' and grade of steel

f_y in N/mm^2	d/d'			
	0.05	0.1	0.15	0.20
250	217	217	217	217
415	355	353	342	329
500	424	412	395	370
550	458	441	419	380

Note: The value of f_{cc} is very small as compared to the value of f_{sc} and can be neglected.

Type of problem:

Type 1: to find out the moment of resistance of a given section

- Total compression = total tension $C_1 + C_2 = T$
- $0.36 f_{ck} b x_u + A_{sc} \cdot (f_{sc}) = 0.87 f_y A_{st}$ Find out x_u
- Find $x_{u,max}$ and type of beam
- If $x_u > x_{u,max}$, the section is over reinforced section, then $x_u = x_{u,max}$
- $M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} \cdot (f_{sc}) (d - d')$

Type 2: to find out reinforcement for flexure for a given section and factored moment

- Find out $M_{u,lim}$ and reinforcement $A_{st,lim}$ for a given section by using the following equation $M_{u,lim} = 0.36 f_{ck} b x_{u,max} (d - 0.42 x_{u,max})$
- $A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,max})}$
- Obtain $M_{u2} = M_u - M_{u,lim}$
- Find compression steel from following equation

$$M_{u2} = A_{sc} \cdot (f_{sc}) \cdot (d - d')$$

$$A_{sc} = \frac{M_{u2}}{f(d - d')}$$

- Corresponding tension steel can be found out by from

$$A_{st2} = \frac{(f_{sc} - f_{cc})}{0.87 f_y}$$

- $A_{st} = A_{stim} + A_{st2}$

Question 1:

Find the factored moment of resistance of a beam section 230mm wide and 460mm effective depth reinforced with 2 nos. of 16mm dia bars as compression reinforcement at an effective cover of 40mm and 4 nos. of 20mm dia bars as tension reinforcement. The materials are M20 grade concrete and Fe250 grade steel.

Solution:

$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250$$

$$N/\text{mm}^2 \cdot d' = 40 \text{ mm}$$

$$m$$

$$A_{sc} = 2 \times 201 = 402 \text{ mm}^2$$

$$A_{st} = 4 \times 314 = 1256 \text{ mm}^2 \cdot C_1 + C$$

$$z = T$$

$$0.36 f_{ck} b x_u + A_{sc} \cdot (f_{sc}) = 0.87 f_y A_{st}$$

$$\text{For } d'/d = 40/460 = 0.08, \text{ next higher value } 0.1$$

$$\text{may be adopted } f_{sc} = 217 \text{ N/mm}^2$$

$$0.36 \times 20 \times 230 x_u + 402 \times 217 = 0.87 \times 250 \times 1256$$

$$1656 x_u = 273180$$

$$87234 = 185946 x_u = 112.29 \text{ mm}$$

$$X_{u \max} = 0.53 d = 0.53 \times 460 = 243.8 \text{ mm}$$

$$x_u < X_{u \max}, \text{ the section is}$$

$$\text{under reinforced section } x_u = 112.29 \text{ mm}$$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} \cdot (f_{sc}) \cdot (d - d')$$

$$= 0.36 \times 20 \times 230 \times 112.29 (460 - 0.42 \times 112.29) + 402 \times 217 (460 - 40)$$

$$= 76.76 + 36.64 = 113.4 \text{ KNm}$$

Question2:

A rectangular beam of size 230 mm wide and 500 mm depth is subjected to a factored moment of 200 Knm. Find the reinforcement for flexure. The materials are M20 grade concrete and Fe415 grade steel.

Solution:

$$M_u = 200 \text{ Knm}$$

$$M_{ulim} = 0.36 f_{ck} b x_{u\max} (d - 0.42 x_{u\max}) \\ = 0.36 \times 20 \times 230 \times 0.48 \times 500 (500 - 0.42 \times 0.48 \times 500) = 158.7 \text{ Knm}$$

$$d' = 50 \text{ mm}$$

$$d'/d = 50/500 = 0.1$$

$$f_{sc} = 353 \text{ N/mm}^2$$

$$M_{u2} = M_u - M_{ulim} = 200 - 158.7 = 41.3 \text{ Knm}$$

$$A_{sc} = \frac{M_{u2}}{f_{sc} (d - d')} = \frac{41.3 \times 10^6}{353 (500 - 50)} = 260 \text{ mm}^2$$

$$A_{st2} = \frac{A_s (f_{sc} - f_{cc})}{0.87 f_y} = \frac{260 \times 353}{0.87 \times 415} = 254.2 \text{ mm}^2$$

$$A_{stim} = \frac{M_{ulim}}{0.87 f_y (d - 0.42 x_{u\max})} = \frac{158.7 \times 10^6}{0.87 \times 415 (500 - 0.42 \times 0.48 \times 500)} = 1101 \text{ mm}^2$$

$$A_{st} = 1101 + 254 = 1355 \text{ mm}^2$$

4 Shear, Bond and Development Length (LSM)

4.1 Nominal shear stress in R.C. section, design shear strength of concrete, maximum shear stress, design of shear reinforcement, minimum shear reinforcement, forms of shear reinforcement.

4.2 Bond and types of bond, bond stress, check for bond stress, development length in tension and compression, anchorage value for hooks 90° bend and 45° bend standards lapping of bars, check for development length.

4.3 Numerical problems on deciding whether shear reinforcement is required or not, check for adequacy of the section in shear. Design of shear reinforcement; Minimum shear reinforcement in beams (Explain through examples only).

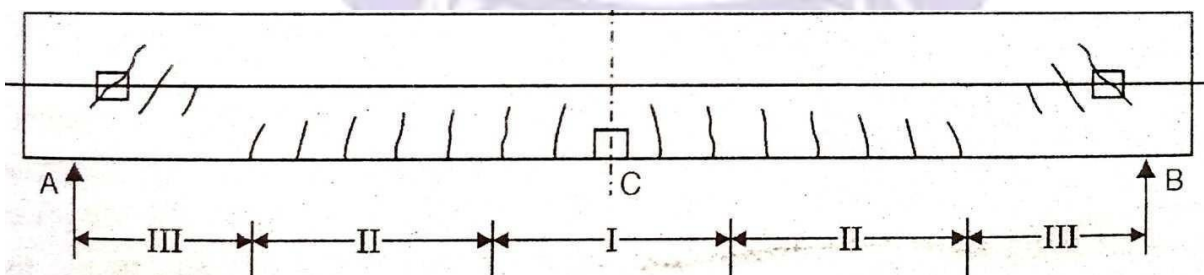
SHEAR STRESS IN REINFORCED CONCRETE BEAMS

When a beam is loaded with transverse loads the Bending Moment (BM) varies from section to section. Shearing stresses in beams are caused by this variation of BM in the beam span. Due to the variation of BM at two sections distance dx apart, there are unequal bending stresses at the same fibre. This inequality of bending stresses produces a tendency in each horizontal fibre to slide over adjacent horizontal fibre causing horizontal shear stress, which is accompanied by complementary shear stress in vertical direction.

SHEAR CRACKS IN BEAMS:-

Under the transverse loading, at any section of the beam, there exists both Bending Moment (BM) and Shear Force (V). Depending upon the ratio of Bending Moment (BM) to Shear Force (V) at different sections, there may be three regions of shear cracks in the beam as follows.

- Region I : Region of flexure cracks.
- Region II : Region of flexure shear cracks.
- Region III : Region of web shear cracks or diagonal tension cracks.



DIFFERENT REGION OF CRACKS IN BEAMS

(a) Region I : Region of flexure Cracks.

This region normally occurs adjacent to mid-span where BM is large and shear force is either zero or very small. The principal planes are perpendicular to beam axis. When the principal tensile stress reaches the tensile strength of the concrete (which is quite low) tensile cracks develop vertically. The cracks are known as flexural cracks resulting primarily due to flexure.

(b) Region II: Region of flexure shear Cracks.

This region is near the quarter span, on both sides, where BM is considerable and at the same time Shear force is significant. The cracks in this region are initiated at the tension face, travel vertically (due to flexure) and gradually tend to develop in the inclined direction towards the Neutral Axis (N.A.), as the shear stress goes on increasing towards the N.A. Since the cracks develop under the combined action of BM and Shear, these cracks are known as flexure-shear cracks.

(c) Region III: Region of web shear Cracks or diagonal tension cracks.

This region is adjacent to each support of the beam where S.F is predominant. Since Shear stress is maximum at the N.A., inclined cracks start developing at the N.A. along the diagonal of an element subject to the action of pure shear. Hence these cracks are known as diagonal tension cracks or web-shear cracks.

Design Shear Strength of Reinforced Concrete

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value (τ_{cmax}) depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

Design shear strength without shear reinforcement (IS 456, cl. 40.2.1)

Table 19 of IS 456 stipulates the design shear strength of concrete τ_c for different grades of concrete with a wide range of percentages of positive tensile steel reinforcement. It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when τ_v is less than τ_c given in Table 3.

100A _s /bd	Concrete grade		
	M20	M25	M30
≤0.15	0.28	0.29	0.29
0.25	0.36	0.36	0.37
0.50	0.48	0.49	0.50
0.75	0.56	0.57	0.59
1.00	0.62	0.64	0.66
1.25	0.67	0.70	0.71
1.50	0.72	0.74	0.76

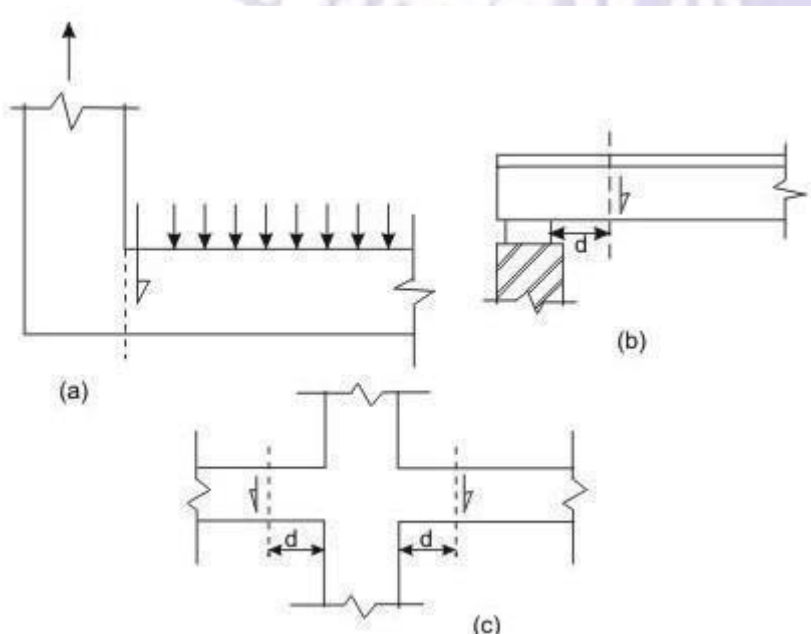
Maximum shear stress with shear reinforcement (cls.40.2.3,40.5.1 and 41.3.1)

Table 20 of IS 456 stipulates the maximum shear stress τ_{cmax} of reinforced concrete beams τ_{cmax} as given below in Table. Under no circumstances, the nominal shear stress in beams τ_v shall exceed τ_{cmax} given in Table for different grades of concrete.

Maximum shear stress τ_{cmax} N/mm²

Concrete grade	M20	M25	M30
τ_{cmax}	2.8	3.1	3.5

Critical Section for Shear



Clauses 22.6.2 and 22.6.2.1 stipulate the critical section for shear and are as follows:

For beams generally subjected to uniformly distributed loads or where the principal load is located further than $2d$ from the face of the support, where d is the effective depth of the beam, the critical sections depend on the conditions of supports as shown in Figs. a, b and c and are mentioned below.

- (a) When the reaction in the direction of the applied shear introduces tension (Fig. a) into the end region of the member, the shear force is to be computed at the face of the support of the member at that section.
- (b) When the reaction in the direction of the applied shear introduces compression into the end region of the member (Figs. b and c), the shear force computed at a distance d from the face of the support is to be used for the design of sections located at a distance less than d from the face of the support. The enhanced shear strength of sections close to supports, however, may be considered as discussed in the following section.

Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c given in Table 19 as recommended in cl. 40.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

The minimum shear reinforcement in the form of stirrups shall be provided such that

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87f_y}$$

Where A_{sv} = total cross-sectional area of stirrup legs effective in

shear S_v = stirrup spacing along the length of the member

b = breadth of the beam or breadth of the web of the flanged beam

f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be greater than 415 N/mm^2

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than $0.75d$ for vertical stirrups and d for inclined stirrups at 45° , where d is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

Design of Shear Reinforcement (cl.40.4 of IS456)

When τ_v is more than τ_c given in Table 19 of IS 456:2000, shear reinforcement shall be provided in any of the three following forms:

- Vertical stirrup
- Inclined stirrup
- Bent up bars along with stirrups

In the case of bent-up bars, it is to be seen that the contribution towards shear resistance of bent-up bars should not be more than fifty percent of that of the total shear reinforcement.

The amount of shear reinforcement to be provided is determined to carry a shear force V_{us} equal to $V_{us} = V_u - r_c b d$

Where b is the breadth of rectangular beams or b_w in the case of flanged beams

The strengths of shear reinforcement V_{us} for the three types of shear reinforcement are as follows:

(a) **Vertical stirrups**, $V_{us} = 0.87 f_y A_{sv} d \times (1/s_{sv})$

(b) **Inclined stirrup**, $V_{us} = \frac{0.87 f_y A_{sv} (\sin \alpha + \cos \alpha)}{s_{sv}}$

(c) **Bent up bars**, $V_{us} = 0.87 f_y A_{sv} \sin \alpha$

Question 1:

A tee beam section having 230 mm width of rib and 460 mm effective depth is reinforced with 5 nos. of 16 mm dia. Bars as tension reinforcement. This section is subjected to a factored shear of 52.5 kN. Check the shear stress and design the shear reinforcement. The materials are M20 grade concrete and Fe415 grade steel. For stirrups mild steel bars may be used.

Solution:

$$V_u = 52.5 \text{ kN}$$

$$A_{st} = 1005.31 \text{ mm}^2$$

$$\text{Nominal shear stress, } r_v = \frac{V_u}{b d} = \frac{52.5 \times 10^3}{230 \times 460} = 0.496 \text{ N/mm}^2$$

$$P_t = \frac{100 A_{st}}{b d} = \frac{100 \times 1005.31}{230 \times 460} = 0.95$$

From table 19 of IS456, $r_c = 0.608 \text{ N/mm}^2$

Here $r_v < r_c$, therefore only nominal shear reinforcement is

required. Select 6 mm dia M.S bars for stirrups

$$A_{sv} = \frac{2 \times \pi \times 6^2}{4} = 56 \text{ mm}^2 \text{ for two legged stirrups}$$

$$\text{For minimum shear reinforcement} = \frac{A_{sv}}{b s_v} \geq 0.4 \frac{r_c}{0.87 f_y}$$

$$\frac{56}{230 \times s_v} \geq \frac{0.4}{0.87 \times 415}$$

$$S_v \leq 132.4 \text{ mm}$$

The spacings shall not exceed

- (a) $0.75d = 0.75 \times 460 = 345 \text{ mm}$
 (b) 300 mm

Provide 6 mm dia. two-legged stirrup @ 130 mm/c

Question 2:

A tee beam section having 230 mm width of rib and 460 mm effective depth is reinforced with 5 nos. of 16 mm dia. Bars as tension reinforcement. The section is subjected to a factored shear of 90 kN. Check the shear stress and design the shear reinforcement. The materials are M20 grade concrete and Fe415 grade steel. For stirrup mild steel bars may be used.

Solution:

$$V_u = 90 \text{ kN}$$

$$A_{st} = 1005.31 \text{ mm}^2$$

$$\text{Nominal shear stress, } r_v = \frac{V_u}{bd} = \frac{90 \times 10^3}{230 \times 460} = 0.85 \text{ N/mm}^2$$

$$P_t = \frac{100 A_{st}}{b d} = \frac{100 \times 1005.31}{230 \times 460} = 0.95$$

From table 19 of IS 456, $r_c = 0.608 \text{ N/mm}^2$

Here $r_v > r_c$, therefore shear reinforcement shall be designed.

$$\text{Shear resistance of concrete} = V_{uc} = r_c b d = 0.608 \times 230 \times 460 \times 10^{-3} = 64.33$$

$$K N V_{us} = V_u - r_c b d = 90 - 64.33 = 25.67 \text{ kN}$$

Using 6 mm dia. two-legged M.S. bars for stirrups

$$A_{sv} = \frac{2 \times \pi \times 6^2}{4} = 56.55 \text{ mm}^2$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 250 \times 56.55 \times 460}{25.67 \times 10^3}$$

$$S_v = 220 \text{ mm}$$

The spacings shall not exceed

- (a) $0.75d = 0.75 \times 460 = 345 \text{ mm}$
 (b) 300 mm

Provide 6 mm dia. two-legged stirrup @ 130 mm/c

Question3:

A tee beam section having 230 mm width of rib and 460 mm effective depth is reinforced with 5 nos. of 16 mm dia. bars as tension reinforcement and out of which 02 nos. bar are bent up at 45°. The section is subjected to a factored shear of 120 KN. Check the shear stress and design the shear reinforcement.

The materials are M20 grade concrete and Fe415 grade steel. For stirrups mild steel bars may be used.

Solution:

$$V_u = 120 \text{ KN}$$

$$\text{Nominal shear stress, } r_v = \frac{V_u}{bd} = \frac{120 \times 10^3}{230 \times 460} = 1.13 \text{ N/mm}^2$$

2 bars are bent up

$$A_{st} = \frac{2 \times \pi \times 16^2}{4} = 402 \text{ mm}^2$$

$$V_{us} = 0.87 f_y A_{sv} \sin \alpha = 0.87 \times 415 \times 402 \times \sin 45^\circ \\ = 102.63 \text{ KN}$$

For the remaining 3 bars

$$\frac{100 A_s}{bd} = \frac{100 \times 3 \times \pi \times 16^2}{4 \times 230 \times 460} = 0.57$$

$$r_c = 0.50 \text{ N/mm}^2$$

$$V_{uc} = r_c b d = 0.50 \times 230 \times 460 \times 10^{-3} = 52.90$$

$$\text{KN } V_{us} = V_u - r_c b d = 120 - 52.90 = 67.10 \text{ KN}$$

Shear resistance provided by bent up bars = $67.10/2 = 33.55 \text{ KN}$

Use 6 mm dia. two-legged stirrups to resist 33.55 KN

$$A_{sv} = \frac{2 \times \pi \times 6^2}{4} = 56 \text{ mm}^2$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 250 \times 56.55 \times 460}{33.55 \times 10^3} = 168.64 \text{ mm}$$

The spacings shall not exceed

$$(a) 0.75d = 0.75 \times 460 = 345 \text{ mm}$$

$$(b) 300 \text{ mm}$$

Provide 6 mm dia. two-legged stirrup @ 130 mm/c

DEVELOPMENT LENGTH

Introduction:

- i) One of the most important assumption in the behavior of reinforced concrete structure is that there is proper bond between concrete and reinforcing bar.
- ii) When a RCC element is loaded, the load is first borne by concrete and is then transferred to steel reinforcement.
- iii) This transfer from concrete to steel can be effected only when there is no relative movement or slip or sliding between them when any one of these two is strained. The force which prevents the slippage between the two constituent material is known as bond.
- iv) When steel bars are embedded in concrete, the concrete after setting adheres to the surface of the bar and thus resists any force that tends to pull or push this rod. The intensity of this adhesive force is called bond stress.
- v) The bond stresses are the longitudinal shear stresses acting on the surface between steel and concrete along its length.
- vi) Bond stress is also known as the interfacial shear. Hence bond stress is the shear stress acting parallel to the reinforcing bar on the interface between the bar and the concrete.

Types of bond:

1. Flexural bond or local bond
2. Anchorage bond or development bond

Flexural bond:

- (i) Flexural bond is one which arises from change in tensile force carried by the bar, along its length, due to change in bending moment along the length of the member. Flexural bond will be more critical at points where shear force is significant.
- (ii) Anchorage bond is that which arises over the length of anchorage provided for a bar. It also arises near the end or cutoff point of a reinforcing bar. The anchorage bond resists the pulling out of the bar if it is in tension or pushing in of the bar if it is in compression.

Anchorage bond stress:

Figure shows a steel bar embedded in concrete and is subjected to a tensile force T . Due to this force there will be a tendency of the bar to slip out and this tendency is resisted by the bond stress developed over the perimeter of the bar along the length.

The required length necessary to develop full

resisting force is called anchorage length in case of axial tension or compression and development length in case of

flexural tension and is designated as L_d .

Hence if ϕ is the nominal diameter of the bar we have

$$(\pi/4)\phi^2 f_y = \tau_{bd} \pi \phi L_d$$

$$L_d = \phi f_y / 4 \tau_{bd}$$

Where f_y is the design stress in steel $= 0.87 f_y L_d = 0.$

$$87 f_y \phi / 4 \tau_{bd}$$

So this indicates that a bar must extend a length L_d beyond any section at which it is required to

develop its full strength so that sufficient bond resistance can be developed.

$$L_d = \phi f_y / 4 \tau_{bd}$$

Where f_y is the design stress in steel $= 0.87 f_y L_d = 0.87 f_y \phi / 4 \tau_{bd}$

$$87 f_y \phi / 4 \tau_{bd}$$

So this indicates that a bar must extend a length L_d beyond any section at which it is required to develop its full strength so that sufficient bond resistance can be developed.

Design bond stress:

- (i) The design bond stress in limit state method for plain bars in tension shall be as given in table.
- (ii) Design bond stress for deformed bars in tension –
For deformed bars these values shall be increased by 60%
- (iii) Design bond stress for bars in compression –
For bars in compression, the values of bond stress for bars in tension shall be increased by 25%.

Standard hooks and bends for end anchorage: Anchorage length:

- (i) The development length required at the end of a bar is known as anchorage length.
- (ii) Space available at the end of the beam is limited to accommodate the full development length (L_d). In that case hooks or bends are provided. The anchorage value (L_e) of hooks or bend is accounted as a contribution to the development length (L_d).
- (iii) The minimum radius specified for a hook is 2ϕ for mild steel bar and 4ϕ for high yield bar.
- (iv) In the case of deformed bars the value of bond stress for various grades of concrete is greater by 60% than the plain bars. Hence the deformed bars may be used without hooks provided anchorage requirements are satisfied.
- (v) The length of straight bars beyond the end of the curves should be at least 4 times the diameter of the bars.

Code requirement for anchoring reinforcing bars:

- (i) **Anchoring bars in tension:** Deformed bars may be used without end anchorages provided development length requirement is satisfied. Hooks should normally be provided for plain bars in tension. The anchorage value of bend shall be taken as 4 times the diameter of bar for each 45° bend subjected to a maximum of 16 times the diameter of the bar. The anchorage value of standard U-type hook shall be equal to 16 times the diameter of the bar.
- (ii) **Anchoring bars in compression:** The anchorage length of straight bar in compression shall be equal to the development length of bars in compression.
- (iii) **Anchoring shear reinforcement :**

Inclined bars – The development length shall be as for bars in tension: this length shall be measured as under:

- (i) In tension zone from the end of the sloping or inclined portion of the bar.
- (ii) In compression zone from mid-depth of the beam

Stirrups-

When the bar is bent through an angle of at least 90° round a bar of atleast it will continued beyond the end of the curve for a length of atleast eight diameter or when the bar is bent through an angle of 135 and is continued beyond the end of the curve for a length of at least six bardiameter or when the bar is bent through an angle of 180 and is continued beyond the end of the curve for a length atleast four bardiameter.

Checking development length of tension bars:

The stress in a reinforcing bar, at every section must be developed on both side of the section. This is done by providing development length L_d to both the sides of the section. Such a development length is usually available at mid span location where positive (or sagging) B.M is maximum for simply supported beams. Similarly such a development length is usually available at the intermediate support of a continuous beam where negative (or hogging) B.M is maximum. Hence no special checking is necessary for such locations. But special checking for development length is essential at following locations;

- i) At simple supports.
- ii) At Cantilever supports.
- iii) At point of contraflexure.
- iv) At point of bar cutoff.

Requirement of development length:

The code stipulates that at the simple support the positive moment tension reinforcement shall be limited to a diameter such that

$$L_d \leq (M_1/V) + L_0$$

Where L_d = development length

M_1 = M.R of the section assuming all reinforcement at the section to be stressed to f_y V = shear force at the section due to design loads

L_0 = sum of anchorage beyond the centre of support and the value of L_0 is limited to d or 12ϕ whichever is greater

The code further recommends that the value of M_1/V may be increased to 30% when the ends of the reinforcement are confined by a compressive reaction. Such situation arises when a beam is simply supported over a wall.

Thus at simple supports where the compressive reaction confines the ends of reinforcing bar, we have

$$L_d \leq 1.3(M_1/V) + L_0$$

Question1:

Calculate the anchorage length in tension and compression for

Single mild steel bar of dia ϕ in concrete of grade M20

An HYSD bar of grade Fe415 of dia ϕ in concrete of grade M20

Solution:

(a) M.S bar

(i) Tension:

$$\text{Design stress for M.S bar} = \sigma_s = 0.87f_y = 0.87 \times 250 = 217.5$$

$$\text{N/mm}^2 \tau_{bd} = 1.2 \text{N/mm}^2$$

$$\text{anchorage length} = \text{development length} \#$$

$$= 0.87f_y \phi / 4\tau_{bd}$$

$$= 217.5 \phi / 4 \times 1.2$$

$$= 45.3 \phi \text{ or } 46 \phi$$

(ii) Compression:

$$\sigma_s = 0.87f_y = 0.87 \times 250 = 217.5 \text{N/mm}^2$$

$$\tau_{bd} = 1.2 \times 1.25 (\text{for compression}) = 1.5$$

$$\text{N/mm}^2 L_d = 0.87f_y \phi / 4\tau_{bd} = 217.5 \phi / 4 \times 1.5$$

$$= 36.3 \phi = 37 \phi$$

(b) HYSD bar

(i) Tension:

$$\sigma_s = 0.87f_y = 0.87 \times 415 = 361 \text{ N/mm}^2$$

$$\tau_{bd} = 1.2 \times 1.6 (\text{HYSD bar}) = 1.92$$

$$\text{N/mm}^2 L_d = 361 \# / 4\tau_{bd} = 217.5 \phi / 4 \times 1.92 = 47 \#$$

(ii) Compression:

$$\sigma_s = 0.87f_y = 0.87 \times 415 = 361 \text{ N/mm}^2$$

$$\tau_{bd} = 1.2 \times 1.6 (\text{HYSD}$$

$$\text{bar}) \times 1.25 (\text{comp}) = 2.4 \text{N/mm}^2 L_d = 361 \# / 4\tau_{bd} = 217.5 \phi / 4 \times 2$$

$$.4 = 37.6 \# = 38 \#$$



Question2:

A simply supported beam is 25 cm by 50 cm and has 2-20 mm HYSD bars going into the support. If the shear force at the centre of support is 110 kN at working loads, determine the anchorage length. Assume M20 mix and Fe415 grade steel.

Solution:

$$\text{Factored S.F.} = 1.5 \times 110 = 165 \text{ kN}$$

$$A_{st} = 2 \times (\pi/4) d^2 = 628 \text{ mm}^2$$

Assuming 25

mm clear cover to the longitudinal bar Effective depth = 5

$$50 - 25 - (20/2) = 465 \text{ mm}$$

$$F_y = 415 \text{ N/mm}^2$$

$$M_1 = 0.87 f_y A_{st} (d -$$

$$0.42 x_u) \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 628}{0.36 \times 20 \times 250} = 125.97 = 126 \text{ mm}$$

$$x_{u\max} = 0.48 d = 0.48 \times 465 = 223.2 \text{ mm}$$

$$x_u < x_{u\max}$$

$$M_1 = 0.87 \times 415 \times 628 (465 - 0.42 \times 126)$$

$$= 93.43 \times 10^6 \text{ Nm}$$

$$\tau_{bd} = 1.2 \times 1.6 = 1.92$$

$$L_d = 0.87 \times 415 \phi / 4 \times 1.92$$

$$= 47 \phi$$

If the bar is given 90° bend at the centre of the support its anchorage value $L_0 = 8\phi = 8 \times 20 = 160$

$$\text{mm } L_d \leq 1.3 (M_1/V) + L_0$$

$$47 \phi \leq \frac{1.3 \times 93.45 \times 10^6}{165 \times 10^3} + 160$$

$$\phi \leq 19 \text{ mm}$$

since actual bar diameter of 20 mm is greater than 19 mm there is a need to increase the anchorage length L_0 to 12ϕ

i.e. 240 mm

$$L_d \leq 1.3 (M_1/V) + L_0$$

$$47 \phi \leq \frac{1.3 \times 93.45 \times 10^6}{165 \times 10^3} + 240$$

$$\phi \leq 20.8 \text{ mm}$$

DESIGN OF BEAMS

Design procedure:

The procedure for design of a beam may be summarized as follows:

- (i) Estimation of load:
 - The correct estimation of loads, a beam has to bear, leads to an economical and safe design of the beams.
 - A designer should not forget to account for any possible load acting on the structure as it leads to an underdesign of the member and subsequent failure of the beam.
 - The dead load on the beam may be self weight of the beam, floor finish, partitions and some special fixed load if specified.
 - The live loads shall be different for different structures depending on the functional use of the structure.
- (ii) Analysis:
 - Using the above determined loads, the shear force and bending moment are found out and diagrams drawn.
- (iii) Design:

After analysis design the beam as follows:

 - Using the maximum moment, calculate the depth of beam required for balanced section.
 - Find out the steel area required for design moment.
 - Check the shear stress and development length of bars.
 - If some bars are curtailed, check for curtailment using curtailment rules.
 - Check the deflection and cracking using rules for control of deflection and cracking.
 - Draw complete sketches of designed beam with elevation and section.

Basic rules for design:

- (i) Effective span:

Simply supported beam or slab: The effective span of a member that is not built integrally with its support shall be taken as clear span plus the effective depth of slab or beam or centre to centre of support whichever is less.

- (ii) Control of deflection:

The deflection of a structure or part thereof shall not adversely affect the appearance or efficiency of the structure or finishes or partitions. For beams and slabs the vertical deflection limit may generally be assumed to be satisfied provided that the span to depth ratios are not greater than the values obtained as below:

- (a) Basic values of span to effective depth ratios for span up to 10m:

Cantilever -----	7
Simply supported -----	20
Continuous	26

- (b) For span above 10 m the value in (a) may be multiplied by a factor $10/\text{span}$ in meter except in case of cantilever.

- (c) Depending on the area and the type of steel for tension reinforcement the values in (a) & (b) shall be multiplied by modification factor as per graph on p.38.

$$F_s = \text{the steel stress of service loads} \\ = 0.58 f_y (\text{area of cross section of steel required} / \text{area of cross section of steel provided})$$

- (iii) Reinforcement in beams:
 (a) Tension reinforcement:

Minimum reinforcement – the minimum area of the tension reinforcement shall not be less than that given by the following:

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y}$$

Maximum reinforcement – the maximum area of tension reinforcement shall not exceed $0.04bD$

- (b) Compression reinforcement:

Maximum reinforcement – the maximum area of compression reinforcement shall not exceed $0.04bD$

- (iv) Criteria for development length:

- According to clause 26.2 of IS:456, the calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length.
- No bar can be bent up or curtailed up to a distance of development length from the point of maximum moment.
- For example for Fe415 grade reinforcement, the development length in concrete of grade M20 in tension is 47ϕ . If 20 mm diameter bar is used, the bars cannot be bent or curtailed up to a distance of $47 \times 20 = 940$ mm from the point of maximum bending moment.
- The code stipulates that at the simple support the positive moment tension reinforcement shall be limited to a diameter such that

$$L_d \leq (M_1/V) + L_0$$

Where L_d = development length

$$M_1 = M.R \text{ of the section assuming all reinforcement at the section to be stressed to } f_{yd}$$

f_{yd}

V = shear force at the section due to design loads

L_0 = sum of anchorage beyond the centre of support and the value of L_0 is limited to 12ϕ whichever is greater

- The code further recommends that the value of M_1/V may be increased to 30% when the ends of the reinforcement are confined by a compressive reaction. Such situation arises when a beam is simply supported over a wall. Thus at simple supports where the compressive reaction confines the ends of reinforcing bar, we have

$$L_d \leq 1.3(M_1/V) + L_0$$

- (v) Slenderness limit for beams:

To ensure lateral stability of

a beam as per clause 23.3 of IS:456 for simply supported and continuous

beam

Clear span $\beta \leq 60b$

$$\beta \leq \frac{250b^2}{d}$$

For cantilever beams

Clear span $\beta \leq 25b$

$$\beta \leq \frac{100b^2}{d}$$

Question1:

A simply supported rectangular beam of 6m span carries a uniformly distributed characteristic load of 24KN/m inclusive of self weight of 24 KN/m. Design the beam. The materials are grade M20 concrete and HYS reinforcement of grade Fe415. The beam is resting on RCC column.

Solution:

Factored load = $1.5 \times 24 = 36 \text{ KN/m}$

$$M_u = \frac{36 \times 6^2}{8} = 162 \text{ KNm}$$

$$V_u = \frac{36 \times 6}{2} = 108 \text{ KN}$$

(a) Calculation of depth:

Assume width of the section $b = 300 \text{ mm}$

$$d_{\text{req}} = \sqrt{\frac{M_u}{Q_{\text{lim}} \times b}} \\ = \sqrt{\frac{162 \times 10^6}{2.76 \times 300}} = 442.32 \text{ mm}$$

Provide effective depth $d = 500 \text{ mm}$

Assume clear cover of 30 mm and 20 mm dia Fe415

bar Overall depth $= D = 500 + 30 + 10 = 540 \text{ mm}$

(b) Calculation of steel area:

$$A_{\text{st}} = \frac{M_u}{0.87 f_y (d - 0.42 x_{\text{u,max}})} = \frac{162 \times 10^6}{0.87 \times 415 (500 - 0.42 \times 0.48 \times 500)} \\ = 1123.97 \text{ mm}^2$$

Provide 4 nos. of 20 mm dia bar giving $A_{\text{st}} = 1256.64 \text{ mm}^2$

Let 2 bars are bent at $1.25D = 1.25 \times 540 = 675 \text{ mm}$ from the face of the support.

The remaining bars should extend within the support for a distance of $L/3 = \frac{47 \times 20}{3} = 313 \text{ mm}$

(c) Check for development length:

(i) A bar can be bent up at a distance greater than $L_d = 47 \phi$ from the centre of the support i.e. $47 \times 20 = 940 \text{ mm}$

In this case the distance is $= 3000 - 940 = 2060 \text{ mm}$ (safe)

(ii) For the remaining bars $A_{\text{st}} = 628 \text{ mm}^2$

$$M_u = 0.87 f_y A_{\text{st}} d \left(1 - \frac{A_{\text{st}} f_y}{b d f_{ck}} \right) \\ = 0.87 \times 415 \times 628 \times 500 \left(1 - \frac{628 \times 415}{300 \times 500 \times 20} \right) \\ = 103.62 \text{ KNm}$$

$$V_u = 108 \text{ KN}$$

$$L_0 = 12 \text{ m (assumed)}$$

As the reinforcement is

confined by compressive reaction $L_d \leq 1.3(M_1/V) + L_0$

$$\frac{\phi \sigma_s}{4c_{bd}} \leq 1.3 \frac{103.62 \times 10^6}{108 \times 10^3} + 12 \times 20$$

$$47\phi \leq 1.3 \frac{103.62 \times 10^6}{108 \times 10^3} + 12 \times 20$$

$$\phi \leq 31.64 \text{ mm}$$

$$\phi_{\text{provided}} = 20 \text{ mm (safe)}$$

The remaining bars should extend within the support for a distance of $l/3 = \frac{47 \times 20}{3} = 313 \frac{1}{3}$ mm. If support width is 300 mm, the bars extend for $150 + L_0 = 150 + (12 \times 20) = 390$ mm within the support.

(d) Check for shear:

$$V_u = 108 \text{ KN}$$

$$\tau_v = \frac{V_u}{bd} = \frac{108 \times 10^3}{300 \times 500} = 0.72 \text{ N/mm}^2$$

$$100 \frac{A_s}{bd} = \frac{100 \times 628}{300 \times 500} = 0.42$$

$$\tau_c = 0.44 \text{ N/mm}^2 < \tau_v$$

hence shear design is necessary. At

support, $V_{us} = V_u - V_{uc}$

$$= (108 \times 10^3) - (0.44 \times 300 \times 500) = 42000 \text{ N}$$

$$= 42 \text{ KN}$$

Capacity of bent up bars to resist shear = $0.87 f_y A_{sv} \sin \alpha$

$$= 0.87 \times 415 \times 628 \times \sin 45^\circ = 160.33 \text{ KN}$$

Bent up bars share 50% = 21

KN Vertical stirrup share 50% = 21 KN

At distance, where bent up bars are not

available $V_{us} = 42 \text{ KN}$

Design stirrup for shear = 42

KN Using 6mm dia mild steel

stirrup $A_{sv} = 56 \text{ mm}^2$

$$V_{us} = \frac{0.87f_v A_s v d}{s_v}$$

$$42 \times 10^3 = \frac{0.87 \times 250 \times 56 \times 500}{s_v}$$

$$s_v = 145 \text{ mm}$$

Spacings should not exceed

(i) $0.75d = 0.75 \times 500 = 375 \text{ mm}$

(ii) 300 mm

(iii) 145 mm

Provide 6 mm dia two-legged mild steel stirrup @ 140 mm/c.

(e) Check for deflection:

Basic span/d ratio =

$$20 \text{ Service stress} = 0.58 f \frac{A_{streg}}{A_{stpro}}$$
$$= 0.58 \times 415 \times \frac{1123.97}{1256.64}$$

$$= 215.29 \text{ N/mm}^2$$

$$100 \frac{A}{bd} = \frac{100 \times 1256.64}{300 \times 500} = 0.84$$

Modification factor = 1.15

$$\text{Span/d permissible} = 20 \times 1.15 = 23$$

$$\text{Actual span/d} = 6000/500 = 12 \quad (\text{safe})$$

5 Analysis and Design of T-Beam (LSM)

5.1 General features, advantages, effective width of flange as per IS: 456-2000 code provisions.

5.2 Analysis of singly reinforced T-Beam, strain diagram & stress diagram, depth of neutral axis, moment of resistance of T-beam section with neutral axis lying within the flange.

5.3 Simple numerical problems on deciding effective flange width. (Problems only on finding moment of resistance of T-beam section when N.A. lies within or up to the bottom of flange shall be asked in written examination)..

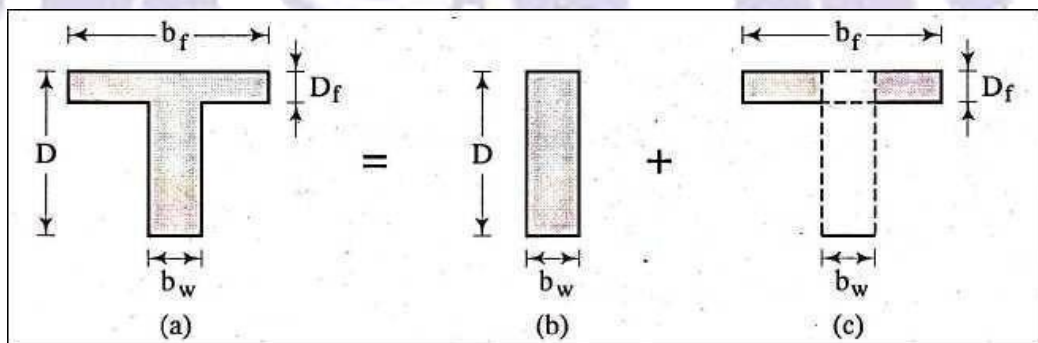
ANALYSIS AND DESIGN OF T-BEAM

Introduction:

- When there is a reinforced concrete slab over a reinforced concrete beam, the slab and beam can be designed and constructed in such a way that they can act together.
- The concrete in the slab, which is on the compression side of the beam (in the middle portion of continuous beam) can be made to resist the compression force and the tension can be carried by the steel in the tension side of the beam. These combined beam and slab units are called "flanged beam".
- They may be T or L beams depending on whether the slab is on both or only on one side of the beam.
- T or L beams act as flanged beams only between the supports where the bending moment is positive and the slab is on the compression side of the beam.
- Over the support where the bending moment is negative the slabs are on the tension side; hence the beam acts only as a rectangular beam with tension steel placed in the slab portion of the beam. Thus at places of negative moment these beams have to be designed as singly or doubly reinforced rectangular beams.

A 'T' beam or 'L' beam can be considered as a rectangular beam with dimensions b_w .

D plus a flange of size $(b_f - b_w) \times D_f$. It is shown in the figure beam (a) is equivalent to beam (b) + beam (c).



The flanged beam analysis and design are analogous to doubly reinforced rectangular beam. In doubly reinforced beams additional compressive is provided by adding reinforcement in compression zone, whereas in flanged beams, this is provided by the slab concrete, where the spanning of the slab is perpendicular to that of beam and slab is in compression zone. If the spanning of the slab is parallel to that of the beam, some portion of slab can be made to span in the direction perpendicular to that of the beam by adding some reinforcement in the slab.

A flanged beam can be also doubly reinforced.

The moment of resistance of a T beam is sum of the moment of resistance of beam (a) is the sum of moment of resistance of beam (b) and moment of resistance of beam (c). Similarly the steel area required for beam (a) shall be equal to the sum of the steel required for the beam (b) and the steel area required for beam (c).

Position of neutral axis:

For a flanged beam, the Neutral Axis either (a) lies in flange or (b) lies in web

(a) Neutral axis lies in flange ($x_u < D_f$)

- When the neutral axis lies in the flange, the size of the compression zone becomes $b_f \times x_u$
- As concrete does not resist any tension, the width of tension zone has no effect on the M. R of the section.
- Therefore this beam can be considered as a rectangular beam of dimension $b_f \times d$ and the formula derived for rectangular beams shall be applied.

- For a singly reinforced rectangular beam:

$$\text{Equating total compression and total tension } \sum X = 0$$

$$.87 f_y A_{st} = \frac{0.36 f_{ck} b x_u}{0.36 f_{ck} b}$$

- To find out the type of the beam, $x_{u,max}$ shall be found out and compared with actual value of neutral axis x_u

If $x_u < x_{u,max}$, the section is under reinforced section

If $x_u > x_{u,max}$, the section is over reinforced section, then $x_u = x_{u,max}$ If

$x_u = x_{u,max}$, the section is balanced section

- For under reinforced

$$\text{section } M_u = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$$

Or

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

- For over reinforced section

$$M_{u,lim} = 0.36 f_{ck} b_f x_{u,max} (d - 0.42 x_{u,max})$$

Question 1:

A tee beam of effective flange width 1200 mm, thickness of slab 100mm, width of rib 300mm and effective depth of 560 mm is reinforced with 4 nos. of 25 mm dia bars. Calculate the factored moment of resistance. The materials are M₂₀ grade concrete and Fe415 grade steel.

Solution:

$$b_f = 1200$$

$$D_f = 100$$

$$b_w = 300$$

$$d = 560$$

m

$$A_{st} = 1964 \text{ mm}^2$$

Assume that N.A. lies in flange ($x_u < D_f$)

$$\text{Total compression} = 0.36 f_{ck} b_f x_u$$

$$\text{Total tension} = 0.87 f_y A_{st}$$

Equating, total compression = total tension

$$0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 1200 x_u = 0.87 \times 415 \times 1964$$

$$x_u = 82.07 \text{ mm} < D_f$$

hence N.A. lies in the

$$\text{flange, } x_{u\max} = 0.48 \times 560 = 268.8$$

mm

$$x_u <$$

$x_{u\max}$, the section is under reinforced section $M_u = 0.3$

$$6 f_{ck} b_f x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 1200 \times 82.07 (560 - 0.42 \times 82.07) = 372.65 \times 10^6 \text{ Nm}$$

$$= 372.65 \text{ Knm}$$

6 Analysis and Design of Slab and Stair case (LSM)

- 6.1 Design of simply supported one-way slabs for flexure check for deflection control and shear.
- 6.2 Design of one-way cantilever slabs and cantilevers chajjas for flexure check for deflection control and check for development length and shear.
- 6.3 Design of two-way simply supported slabs for flexure with corner free to lift.
- 6.4 Design of dog-legged staircase
- 6.5 Detailing of reinforcement in stairs spanning longitudinally

ANALYSIS AND DESIGN OF SIMPLY SUPPORTED SLAB

Slab-Slabs are plate elements having the depth D much smaller than its span and width. They usually carry a uniformly distributed load and form the floor or roof of the building. They are generally of two types:-

(1) **Onewayspanning slab-**

- (i) The slab supported on all four edges but $l_y/l_x > 2$. Here as l_y is much more than l_x then there will be a tendency of the slab to bend in one direction (about l_x) only. Hence the slab where $l_y/l_x > 2$ is called one way slab provided that it is supported on all four edges.
- (ii) The slab supported on two opposite supports is a one way spanning slab.

(2) **Two way spanning slab-**

If the slab is supported on all four edges and if $l_y/l_x \leq 2$, the tendency of the slab is to bend in both directions. Such slabs are called two way spanning slab. The following conditions should be satisfied

- (i) The slab shall be supported on all four edges.
- (ii) $l_y/l_x \leq 2$.

Analysis of one way slab:-

The analysis of the slab spanning in one direction is done by assuming it to be a beam of l m width.

In addition to the main reinforcement, transverse reinforcement (also called distribution reinforcement) is also provided in a direction at right angles to the main reinforcement. The transverse reinforcement is provided to serve the following purpose-

- (i) It resists the temperature and shrinkage stresses.
- (ii) It keeps the main reinforcement in position.
- (iii) It distributes the concentrated and non-uniform load throughout the slab more evenly and uniformly.

Basic rules for design:-

(1) Effective span – Thus in case of freely supported slab the effective span is taken equal to the distance between centre to centre of supports or the clear distance between the supports plus the effective depth of the slab whichever is less.

- (2) Control of deflection- The basic values of span to effective depth
- | | |
|------------------|----|
| Cantilever | 7 |
| Simply supported | 20 |
| Continuous | 26 |

(3) Reinforcement Requirements:

- (i) Minimum reinforcement-
The mild steel reinforcement in either direction in slab shall not be less than 0.15% of the total cross-sectional area.
- (ii) For HYSD bar this value is 0.12% of the total cross-sectional area.

- (iii) Maximum diameter- The diameter of the bar shall not exceed one eighth of the total thickness of the slab.
- (iv) Minimum diameter- SP:34 gives the guideline for minimum diameter of bars in slabs. Accordingly, for main bars the minimum diameter shall be 10 mm for plain bars and 8 mm for deformed bars. For distribution bars, the minimum diameters shall be 6 mm in any case.
- (4) Shear Stress- In normal cases the shear in slabs is not critical. However shear shall be checked in accordance with the code requirements of clause 40.2. For solid slab the design shear strength in concrete shall be τ_c .
- (5) Deflection- This shall be checked in the same manner as the beams. The slabs are thin elements and deflection may govern the thickness of the slab.
- (6) Cracking- To ensure that the cracking of the slabs is not excessive, spacing of the reinforcement shall be limited to the followings:
 For main bars, spacing $\beta 3d$
 $\beta 300 \text{ mm}$ for secondary bars, spacing $\beta 5d$
 $\beta 450$
 mm Where d = Effective depth of slab.
- (7) Cover: For mild exposure, specified clear cover is 20mm. This can be reduced by 5mm when the reinforcement of 12mm diameter or less is used.
- (8) Development Length: The development length in the slab shall be checked in the same manner as for beams.
- For checking development length, L_0 may be assumed as 8ϕ for HYSD bars (usually end anchorage is not provided) and 12ϕ for mild steel.

Question 1:

A simply supported one way slab of clear span 3.0 m is supported on masonry walls of thickness 350mm. slab is used for residential loads. Design the slab. The materials are grade M20 concrete and HYSD reinforcement of grade Fe415. Live load shall be 2 KN/m^2 .

Solution:

Assume overall depth of slab $D = 130 \text{ mm}$

(a) Load calculation:

$$\begin{aligned} \text{Dead load} &= 0.13 \times 25 = 3.25 \\ \text{KN/m}^2 \text{ Floor finish} &= 1.00 \text{ KN/m}^2 \\ \text{Live load} &= 2.00 \text{ KN/m}^2 \end{aligned}$$

$$\text{Total load} = 6.25$$

$$\text{KN/m}^2 \text{ Factored load} = 1.5 \times$$

$$6.25 = 9.4 \text{ KN/m}^2$$

$$\begin{aligned} \text{Effective span} &= (1) 3000 + 350 = 3350 \text{ mm c/c support} \\ &= 3000 + 110 (\text{effective depth}) = \end{aligned}$$

$$3110 \text{ mm Use } 3.11 \text{ m effective span.}$$

(b) Calculation of bending moment and shear force: Co

consider 1 m length of slab

Load = 9.4 kN/m

Maximum moment = $9.4 \times 3.11^2 / 8 = 11.36$

kNm. Maximum shear = $9.4 \times 3 / 2 = 14.1$ kN (based on clear span)

(c) Calculation of depth:

$$d_{\text{req}} = \sqrt{\frac{M_u}{Q_{\text{lim}} \times b}}$$
$$= \sqrt{\frac{11.36 \times 10^6}{2.76 \times 1000}} = 64.15 \text{ mm}$$

$$d_{\text{provided}} = 130 - 15 (\text{cover}) - 5 (\text{assume } 10\phi \text{ bar})$$
$$= 110 \text{ mm}$$

(d) Calculation of steel area:

$$A_{\text{st}} = \frac{M_u}{0.87 f_y (d - 0.42 x_{\text{u,max}})} = \frac{11.36 \times 10^6}{0.87 \times 415 (110 - 0.42 \times 0.48 \times 110)}$$
$$= 358 \text{ mm}^2$$

$$\text{Spacing} = \frac{\text{area of one bar} \times 1000}{\text{required area in mm}^2 \text{ per metre}}$$
$$= \frac{\frac{\pi \times 8^2}{4} \times 1000}{358} = 140 \text{ mm}$$

Provide 8 mm dia bar @ 140 mm/c giving $A_{\text{st}} = \frac{\text{area of one bar} \times 1000}{\text{spacing}} = 4 \times \frac{\frac{\pi \times 8^2}{4} \times 1000}{140} = 360 \text{ mm}^2$

Half

the bars are bent at $0.11 = 0.1 \times 3100 = 310$ mm. Remaining bars

provide 180 mm^2 area.

$$p_r = \frac{100 A_s}{b D} = \frac{100 \times 180}{1000 \times 130} = 0.13 > 0.12$$

i.e. remaining bars provide minimum steel. Thus half bars may be bent up.

Distribution steel = $\frac{0.15 \times 1000 \times 130}{100} = 195 \text{ mm}^2$, using mild steel

$$\text{Spacing} = \frac{\text{area of one bar} \times 1000}{\text{required area in mm}^2 \text{ per metre}}$$

$$= \frac{\frac{\pi \times 6^2}{4} \times 1000}{195} = 145 \text{ mm}$$

Provide 6 mm ϕ @ 140 mm/c

(e) Check for shear: F_o

bars at support

$$\text{Correct } d = 130 - 15 - 4 \\ = 111 \text{ mm}$$

$$p_t = \frac{100A_s}{bd} = \frac{100 \times 180}{1000 \times 111} = 0.16$$

for slab upto 150 mm thickness, $k =$

$$1.3 \text{ value of } f_c \text{ from IS:456} = 0.28 \text{ N/mm}^2$$

$$\text{design shear strength} = k\tau_c = 1.3 \times 0.28 = 0.364 \text{ N/mm}^2$$

$$\text{actual shear stress} = \tau_v = \frac{14.1 \times 10^3}{1000 \times 111} = 0.13 \text{ N/mm}^2 < 0.364 \text{ N/mm}^2 \quad (\text{safe})$$

(f) Check for development length:

For continuing bars, $A_{st} = 180 \text{ mm}^2$. Also, the ends of the reinforcement are confined by compressive reaction.

$$M_{ul} = 0.87 f_y A_{st} \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right) \\ = 0.87 \times 415 \times 180 \times 111 \left(1 - \frac{180 \times 415}{1000 \times 111 \times 20} \right) \\ = 7$$

$$\text{KNm } V_u = 14.1 \text{ KN}$$

$$L_0 = 12 \neq$$

$$(\text{assumed}) L_d \leq 1.3 \left(\frac{M_1}{V} + L_0 \right)$$

$$M_1/V + L_0$$

$$\frac{\phi \sigma}{4 c b d} \leq 1.3 \frac{7 \times 10^6}{14.1 \times 10^3} + 12 \times 8$$

$$47 \phi \leq 1.3 \frac{7 \times 10^6}{14.1 \times 10^3} + 6$$

$$\phi \leq 15.77 \text{ mm}$$

$$\phi_{\text{provided}} = 8 \text{ mm} \quad (\text{safe})$$

(g) Check for deflection:

Basic span/d ratio =

$$20 \text{ Service stress} = 0.58 f_{st,reg} \\ = 0.58 \times 415 \times \frac{358}{360}$$

$$= 240 \text{ N/mm}^2$$

$$\frac{100A}{sbd} = \frac{100 \times 360}{1000 \times 111} = 0.32$$

Modification factor = 1.5

Span/d permissible = $20 \times 1.5 = 30$

Actual span/d = $3100/111 = 28 < 30$ (safe)



